Missouri Collegiate Mathematics Competition

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The tenth annual Missouri Collegiate Mathematics Competition was held in conjunction with the spring meeting of the Missouri Section of the MAA on the campus of Missouri Western State College in April. The contest is sponsored by the Missouri Section and is held at the site of the spring meeting. After ten years, the competition continues to be quite popular with students of Missouri colleges and universities, and, of course, the level of student participation in the state meeting has increased dramatically.

Ten years ago when the contest was in it incubation, it was reasoned that to appeal to the interests of students and institutions of higher learning at all levels, the examination questions should range in difficulty from the calculus and discrete mathematics level to problems approaching the difficulty of those on the Putnam Exam. It was further decided that the contest would consist of two two-and-one-half hour sessions, the first to be held from 7:30 pm to 10:00 pm on the Thursday evening prior to the first day of the spring meeting, and the second on Friday morning from 8:30 am to 11:00 am. Competition would be among teams of up to three students, and a college or university could enter two official teams to be accompanied by one or more sponsors from their institution. At the request of a number of colleges and universities, unofficial teams were later allowed to enter, but they were ineligible for awards.

The first two questions in each session are of the type mentioned above and are intended to be solvable by all teams in the competition. The following are examples of these questions: one is a traditional parabola question and the other is a discrete mathematics question.

Let $P \neq (0,0)$ be a point of the parabola $y = x^2$. The normal line to the parabola at P will intersect the parabola at another point, say Q. Find the coordinates of P so that the area bounded by the normal line and the parabola is a minimum.

The numbers ± 1, ± 2, ..., ± 2004 are written on a blackboard. You decide to pick two numbers x and y at random, erase them, and write their product, xy, on the board. You continue this process until only one number remains. Prove that the last number is positive.

In addition, there is typically a number theory question on the test. The following is an example:

Find all integer solutions (x, y) to the equation $xy = 5x + 11y$.

The last question is typically a challenging analysis-type question. The following is an example:

Prove that in the MacLaurin series for $\tan \theta$, $-\pi/2 < \theta < \pi/2$, every coefficient is non-negative.

The contest is governed by a committee of seven college and university mathematics faculty members from around the state. The committee members submit questions to the chair and once per year they meet to finalize the exam. Tests for the coming contest and the following year are prepared at the meeting. Questions need not be original, but they very often are. Some may be found in journals and problem books, and some are modifications of such problems.

The competition is held in a room which will accommodate approximately 30 teams which participate annually. Each team has its own table, each student receives a copy of the exam, and scratch paper is provided as needed. Snacks and drinks are available to the students during each session. At the end of each session, the solutions which are secretly coded, are collected, separated by problem number and graded by the committee members. One grader is responsible for assessing all the solutions for a given problem number, and zero to 10 points are assigned to each team. The results are reported to the scorer, and he only knows where the teams rank relative to each other. The results of the two sessions are combined to determine the top three teams and the ranking of all teams.

The interest in the competition among institutions of all levels has been fairly constant over the past 10 years. Many four-year colleges, all the state universities, and the large research based universities participate annually. One would anticipate that the later institutions would dominate the awards, but that has not always been

the case. Often one very good student can carry his team to a high ranking and even first place.

The expenses for an institution's entry in the competition and for the student's travel and boarding are borne by the institution. The contest entry fee is intended to cover the cost of materials, duplicating, and two meals for the students.

The team achieving the highest total score for the two sessions receives a traveling trophy which their department displays for the year prior to the next contest. In addition, each team member receives a plaque indicating that she/he was a member of the winning team for the indicated year. Each participant in the competition receives a certificate of participation, and the awards are presented during the section meeting banquet on Friday evening.

As mentioned above, snacks and food are made available to the students during their participation. To encourage the competitors to mingle and get acquainted, a pizza party is given following the second session. In addition, bowling lanes are reserved for friendly competition among teams that wish to participate after the banquet. A photographer takes pictures of the teams in competition, and a group photograph is shot prior to the pizza party. The pictures are uploaded to the contest webpage before the end of the meeting.

For those who are interested, the contest rules and pictures are available at the following web address:

$$\text{http://www.math-cs.cmsu.edu/}\sim\text{curtisc/contest}$$

The popularity of the competition has exceed the expectations of the organizers. A department chair at one of the participating universities has observed that the competition is the most significant activity of the Missouri Section.

Questions concerning the competition may be directed to Prof. Curtis Cooper who serves as chair of the contest committee. His email address is cnc8851@cmsu2.cmsu.edu.

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