The Discovery of the 43rd and 44th Mersenne Primes at UCM

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August 10, 2012

1. **Mersenne Primes**
2. Lucas-Lehmer Test
3. GIMPS
   - GIMPS
   - GIMPS People
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4. 43rd, 44th, and 47th Mersenne Primes
   - $2^{30402457} - 1$
   - $2^{32582657} - 1$
   - $2^{43112609} - 1$
5. Top 10

Prime Numbers

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- Prime Numbers Less Than 100:

$$2,\ 3,\ 5,\ 7,\ 11,\ 13,\ 17,\ 19,\ 23,\ 29,\ 31,\ 37,\ 41,$$
$$43,\ 47,\ 53,\ 59,\ 61,\ 67,\ 71,\ 73,\ 79,\ 83,\ 89,\ 97$$

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- Examples of Mersenne numbers are:

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\begin{aligned}
3 &= 2^2 - 1 \\
7 &= 2^3 - 1 \\
31 &= 2^5 - 1 \\
127 &= 2^7 - 1 \\
2047 &= 2^{11} - 1
\end{aligned}
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- $2047 = 2^{11} - 1 = 23 \times 89$.

Marin Mersenne

- Mersenne primes are named after a 17th-century French monk and mathematician

Marin Mersenne (1588-1648)

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Definition

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$$S_{n+1} = S_n^2 - 2 \text{ for } n \geq 1.$$

- The first few terms of the S sequence are:

$$4,\ 14,\ 194,\ 37634,\ 1416317954,\ 2005956546822746114,\ 4023861667741036022825635656102100994, \ldots$$

Lucas-Lehmer Test

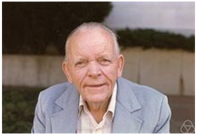
Let p be a prime number. Then

$$M_p = 2^p - 1 \text{ is prime}$$

if and only if

$$S_{p-1} \bmod M_p = 0.$$

Lucas



Lehmer

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Theorem

$M_7 = 2^7 - 1 = 127$ *is prime.*

Proof

i	$S_i \bmod 127$
1	4
2	$(4^2 - 2) = 14 \bmod 127 = 14$
3	$(14^2 - 2) = 194 \bmod 127 = 67$
4	$(67^2 - 2) = 4487 \bmod 127 = 42$

Theorem

$M_7 = 2^7 - 1 = 127$ *is prime.*

Proof

i	$S_i \bmod 127$
1	4
2	$(4^2 - 2) = 14 \bmod 127 = 14$
3	$(14^2 - 2) = 194 \bmod 127 = 67$
4	$(67^2 - 2) = 4487 \bmod 127 = 42$
5	$(42^2 - 2) = 1762 \bmod 127 = 111$

Theorem

$M_7 = 2^7 - 1 = 127$ is prime.

Proof

i	$S_i \bmod 127$
1	4
2	$(4^2 - 2) = 14 \bmod 127 = 14$
3	$(14^2 - 2) = 194 \bmod 127 = 67$
4	$(67^2 - 2) = 4487 \bmod 127 = 42$
5	$(42^2 - 2) = 1762 \bmod 127 = 111$
6	$(111^2 - 2) = 12319 \bmod 127 = 0$

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The Great Internet Mersenne Prime Search

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- George Woltman founded GIMPS in January 1996 and wrote the prime testing software.
- Scott Kurowski wrote the PrimeNet server that supports GIMPS. In 1997 he founded Entropia, a distributed computing software company.

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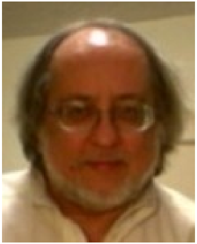
- The GIMPS project consists of 88,074 users, 539 teams, and 642,683 CPUs.
- UCM has over 1000 computers performing LL-tests on Mersenne numbers.

GIMPS People

Woltman

Kurowski



Crandall

GIMPS Links

- The GIMPS home page can be found at: http://www.mersenne.org

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- A Mersenne Prime discussion forum can be found at: http://www.mersenneforum.org

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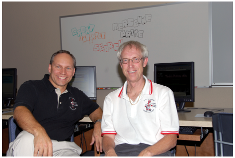
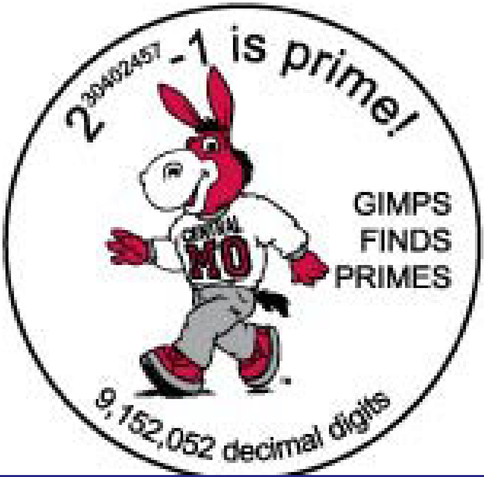
	exponent	Digits in M_p	year	discoverer
43?	30402457	9152052	2005	Cooper, Boone, UCM, GIMPS
44?	32582657	9808358	2006	Cooper, Boone, UCM, GIMPS
47?	43112609	12978189	2008	Smith, UCLA, GIMPS



$2^{30402457}-1$ Button



News About $2^{30402457} - 1$

- On December 15, 2005 at 8:46:58 am (CST), computer commwd102–07l in the Communications Lab (Wood 102) proved that $2^{30402457} - 1$ is prime.

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- News items on the web regarding M30402457 can be found at: http://www.math-cs.ucmo.edu/∼curtisc/M30402457.html

$2^{32582657} - 1$ Button

News About $2^{32582657} - 1$

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- Comments about M30402457 and M32582657 can be found at: http://primes.utm.edu/bios/code.php?code=G9

News About $2^{43112609} - 1$

- On August 23, 2008 in a computer lab in the Mathematics Department at UCLA, Edson Smith and his UCLA team proved that $2^{43112609} - 1$ is prime.

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- Information about M43112609 can be found at: http://www.math.ucla.edu/∼edson/prime/

More News About $2^{43112609} - 1$

Because M43112609 was the first known ten million digit prime number, the Electronic Frontier Foundation (EFF) awarded \$100,000 to GIMPS for this discovery. According to the agreement of GIMPS volunteers, \$50,000 went to Edson Smith and the Mathematics Department at UCLA. \$25,000 went to a charity designated by George Woltman. And the remaining \$25,000 was split among the GIMPS individuals/groups who had found Mersenne primes between one and ten million digits. Since UCM had found two such primes, we received \$6,666 from GIMPS. The UCM money was distributed to colleges and units at UCM based on the percentage of computers running the Mersenne prime program in the college or unit.

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Top 10 Reasons to Search for Large Mersenne Primes

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7. To discover new and more efficient algorithms for testing the primality of large numbers.

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4. To learn more about the distribution of Mersenne primes.

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1. To win the $150,000 offered by the Electronic Frontier Foundation (EFF) for the discovery of the first one-hundred million digit prime number. EFF's motivation is to encourage research in computational number theory related to large primes.

Email Address and Talk URL

Curtis Cooper's Email:
cooper@ucmo.edu

Talk:
http://www.math-cs.ucmo.edu/~curtisc/talks/gimps3/Mersenne6.pdf