The 48th Mersenne Prime, GIMPS, the LL Test, and Perfect Numbers

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University of Central Missouri

July 22, 2013

1. **Mersenne Primes**
   - Primes
   - Mersenne Primes
2. **History of Mersenne Primes**
   - Marin Mersenne
   - Edouard Lucas
   - Computer Era
3. **48th Mersenne Prime**
   - News on 48th Mersenne Prime
4. **GIMPS**
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5. **Lucas-Lehmer Test and Fast Fourier Transforms**
   - Lucas-Lehmer Test

- $2^{11} - 1$ is not prime

Prime Numbers

- A **prime number** is an integer, greater than 1, which has exactly two factors, itself and one.

Prime Numbers

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- Prime Numbers Less Than 100:

$$2,\ 3,\ 5,\ 7,\ 11,\ 13,\ 17,\ 19,\ 23,\ 29,\ 31,\ 37,\ 41,$$
$$43,\ 47,\ 53,\ 59,\ 61,\ 67,\ 71,\ 73,\ 79,\ 83,\ 89,\ 97$$

Mersenne Numbers

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- Examples of Mersenne numbers are:

$$M2 = 3 = 2^2 - 1$$
$$M3 = 7 = 2^3 - 1$$
$$M5 = 31 = 2^5 - 1$$
$$M7 = 127 = 2^7 - 1$$
$$M11 = 2047 = 2^{11} - 1$$

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- $2047 = 2^{11} - 1 = 23 \times 89.$

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Marin Mersenne

- Mersenne primes are named after a 17th-century French monk and mathematician

Marin Mersenne (1588-1648)

Marin Mersenne

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Marin Mersenne

- Mersenne compiled what was supposed to be a list of Mersenne primes with exponents up to 257.
- His list was largely incorrect, as Mersenne mistakenly included M67 and M257 (which are composite), and omitted M61, M89, and M107 (which are prime).

Edouard Lucas

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- Without finding a factor, Lucas demonstrated that M67 is actually composite.

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- On the other side of the board, he multiplied 193,707,721 times 761,838,257,287 and got the same number, then returned to his seat (to applause) without speaking.

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- On the other side of the board, he multiplied 193,707,721 times 761,838,257,287 and got the same number, then returned to his seat (to applause) without speaking.
- A correct list of all Mersenne primes in this number range was completed and rigorously verified only about three centuries after Mersenne published his list.

Computer Era

- The search for Mersenne primes was revolutionized by the introduction of the electronic digital computer.

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- Later Landon Curt Noll found M23209.

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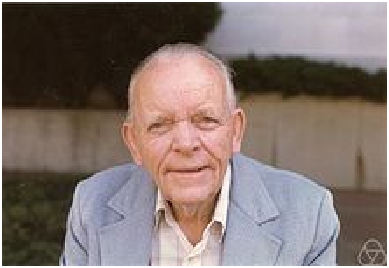

Derrick Henry Lehmer

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- M44497 is the first with more than 10,000 digits.
- M6,972,593 was the first prime with at least 1,000,000 digits.
- All three were the first known prime of any kind of that size.

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- The prime was found on a Dell OptiPlex 745 on August 23, 2008. This is the eighth Mersenne prime discovered at UCLA.
- UCM's part of the prize, for discovering Mersenne primes in December 2005 and September 2006, was 6666 dollars.

- List of 48 Known Mersenne Primes - http://en.wikipedia.org/wiki/Mersenne_prime

News About 48th Mersenne Prime

- Official Press Release
  http://www.mersenne.org/various/57885161.htm

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More About 48th Mersenne Prime

- Fox 4 Kansas City News Story http://fox4kc.com/2013/02/08/ucm-professors-big-prime-number-discovery-has-bragging-rights/

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- Lee Judge Cartoon http://www.cartoonistgroup.com/subject/The-Judge-Comics-and-Cartoons.php

Mersenne Buttons

- M30402457 Button http://www.math-cs.ucmo.edu/∼curtisc/photos/M30402457.jpg

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- M30402457 Button http://www.math-cs.ucmo.edu/∼curtisc/photos/M30402457.jpg
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- M32582657 Button http://www.math-cs.ucmo.edu/∼curtisc/photos/M32582657.jpg
- M57885161 Button http://www.math-cs.ucmo.edu/∼curtisc/images/1.jpg

Jumping GIFS

- 3 Primes GIF
  http://www.math-cs.ucmo.edu/∼curtisc/images/6.gif

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- 3 Primes GIF
  http://www.math-cs.ucmo.edu/∼curtisc/images/6.gif
- UCM GIF
  http://www.math-cs.ucmo.edu/∼curtisc/images/14.gif

Digits of M57885161

- Digits of M57885161 http://www.isthe.com/chongo/tech/math/digit/m57885161/prime-c.html

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- Pronunciation of M57885161
  http://www.isthe.com/chongo/tech/math/digit/m57885161/prime-d.html

The Great Internet Mersenne Prime Search

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- George Woltman founded GIMPS in January 1996 and wrote the prime testing software.
- Scott Kurowski wrote the PrimeNet server that supports GIMPS. In 1997 he founded Entropia, a distributed computing software company.

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- The GIMPS project consists of 98,980 users, 574 teams, and 730,562 CPUs.

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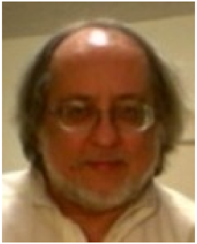
- As of February 5, 2013, GIMPS had a sustained throughput of approximately 129 trillion floating-point operations per second).
- The GIMPS project consists of 98,980 users, 574 teams, and 730,562 CPUs.
- UCM has over 1000 computers performing LL-tests on Mersenne numbers.

GIMPS People

Woltman

Kurowski



Crandall

GIMPS Links

- The GIMPS home page can be found at: http://www.mersenne.org

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- A Mersenne Prime discussion forum can be found at: http://www.mersenneforum.org

Lucas-Lehmer Test

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Definition

Let $S_1 = 4$ and

$$S_{n+1} = S_n^2 - 2 \text{ for } n \geq 1.$$

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Definition

Let $S_1 = 4$ and

$$S_{n+1} = S_n^2 - 2 \text{ for } n \geq 1.$$

- The first few terms of the S sequence are:

  $4,\ 14,\ 194,\ 37634,\ 1416317954,\ 2005956546822746114,$
  $4023861667741036022825635656102100994, \ldots$

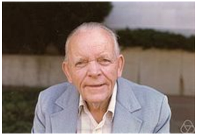
Lucas-Lehmer Test

Let p be a prime number. Then

$$M_p = 2^p - 1 \text{ is prime}$$
$$\text{if and only if}$$
$$S_{p-1} \bmod M_p = 0.$$

Lucas-Lehmer Test


Lucas


Lehmer

Theorem

$M_{11} = 2^{11} - 1 = 2047$ *is not prime.*

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Proof

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2	$(4^2 - 2) = 14 \bmod 2047 = 14$

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i	$S_i \bmod 2047$
1	4
2	$(4^2 - 2) = 14 \bmod 2047 = 14$
3	$(14^2 - 2) = 194 \bmod 2047 = 194$

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i	$S_i \bmod 2047$
1	4
2	$(4^2 - 2) = 14 \bmod 2047 = 14$
3	$(14^2 - 2) = 194 \bmod 2047 = 194$
4	$(194^2 - 2) = 37634 \bmod 2047 = 788$

Theorem

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i	$S_i \bmod 2047$
1	4
2	$(4^2 - 2) = 14 \bmod 2047 = 14$
3	$(14^2 - 2) = 194 \bmod 2047 = 194$
4	$(194^2 - 2) = 37634 \bmod 2047 = 788$
5	$(788^2 - 2) = 620942 \bmod 2047 = 701$

$2^{11} - 1$ is not prime

Proof cont.

i	$S_i \bmod 2047$

$2^{11} - 1$ is not prime

Proof cont.

i	$S_i \bmod 2047$
6	$(701^2 - 2) = 491399 \bmod 2047 = 119$

$2^{11} - 1$ is not prime

Proof cont.

i	$S_i \bmod 2047$
6	$(701^2 - 2) = 491399 \bmod 2047 = 119$
7	$(119^2 - 2) = 14159 \bmod 2047 = 1877$

$2^{11} - 1$ is not prime

Proof cont.

i	$S_i \bmod 2047$
6	$(701^2 - 2) = 491399 \bmod 2047 = 119$
7	$(119^2 - 2) = 14159 \bmod 2047 = 1877$
8	$(1877^2 - 2) = 3523127 \bmod 2047 = 240$

$2^{11} - 1$ is not prime

Proof cont.

i	$S_i \bmod 2047$
6	$(701^2 - 2) = 491399 \bmod 2047 = 119$
7	$(119^2 - 2) = 14159 \bmod 2047 = 1877$
8	$(1877^2 - 2) = 3523127 \bmod 2047 = 240$
9	$(240^2 - 2) = 57598 \bmod 2047 = 282$

$2^{11} - 1$ is not prime

Proof cont.

i	$S_i \bmod 2047$
6	$(701^2 - 2) = 491399 \bmod 2047 = 119$
7	$(119^2 - 2) = 14159 \bmod 2047 = 1877$
8	$(1877^2 - 2) = 3523127 \bmod 2047 = 240$
9	$(240^2 - 2) = 57598 \bmod 2047 = 282$
10	$(282^2 - 2) = 79522 \bmod 2047 = 1736$

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$M_{31} = 2^{31} - 1 = 2147483647$ *is prime.*

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i	$S_i \bmod 2^{31} - 1$

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3	194
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Theorem

$M_{31} = 2^{31} - 1 = 2147483647$ *is prime.*

Proof.

i	$S_i \bmod 2^{31} - 1$
1	4
2	14
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4	37634
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6	669670838

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$M_{31} = 2^{31} - 1 = 2147483647$ *is prime.*

Proof.

i	$S_i \bmod 2^{31} - 1$
1	4
2	14
3	194
4	37634
5	1416317954
6	669670838
7	1937259419

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$M_{31} = 2^{31} - 1 = 2147483647$ *is prime.*

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i	$S_i \bmod 2^{31} - 1$
1	4
2	14
3	194
4	37634
5	1416317954
6	669670838
7	1937259419
8	425413602

$2^{31} - 1$ is prime

i	$S_i \bmod 2^{31} - 1$
9	842014276
10	12692426
11	2044502122
12	1119438707
13	1190075270
14	1450757861
15	877666528
16	630853853
17	940321271
18	512995887
19	692931217

$2^{31} - 1$ is prime

i	$S_i \bmod 2^{31} - 1$
20	1883625615
21	1992425718
22	721929267
23	27220594
24	1570086542
25	1676390412

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26	1159251674
27	211987665
28	1181536708
29	65536
30	0

Fast Fourier Transforms

- Fast Fourier Transform Paper
  http://www.math-cs.ucmo.edu/ curtisc/M57885161.html

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Theorem

If p is an odd prime, then any prime q that divides $2^p - 1$ must be 1 plus a multiple of $2p$. This holds even when $2^p - 1$ is prime

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- $2^5 - 1 = 31$ is prime and $31 = 1 + 3 \times 2 \times 5$.

Theorem

If p is an odd prime, then any prime q that divides $2^p - 1$ must be 1 plus a multiple of $2p$. This holds even when $2^p - 1$ is prime

- $2^5 - 1 = 31$ is prime and $31 = 1 + 3 \times 2 \times 5$.
- $2^{11} - 1 = 2047 = 23 \times 89$, where $23 = 1 + 2 \times 11$ and $89 = 1 + 4 \times 2 \times 11$.

Theorem

If p is an odd prime, then any prime q that divides $2^p - 1$ must be congruent to $\pm 1 \pmod 8$

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- Primes 23 and 89 divide $2^{11} - 1 = 2047$. $23 \equiv 1 \pmod 8$ and $89 \equiv 1 \pmod 8$.

Theorem

If p is an odd prime, then any prime q that divides $2^p - 1$ must be congruent to $\pm 1 \pmod 8$

- Primes 23 and 89 divide $2^{11} - 1 = 2047$. $23 \equiv 1 \pmod 8$ and $89 \equiv 1 \pmod 8$.
- Primes 47 and 178481 divide $2^{23} - 1 = 8,388,607$. $47 \equiv -1 \pmod 8$ and $178481 \equiv 1 \pmod 8$.

Theorem

If M_p is prime, then p is prime.

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Proof

By contradiction. Suppose p is composite. Then $p = ab$ for some $a, b > 1$. But then

$$\begin{aligned} 2^p - 1 = 2^{ab} - 1 &= (2^a)^b - 1 \\ &= (2^a - 1) \cdot (2^{a(b-1)} + 2^{a(b-2)} + \cdots + 2^a + 1). \end{aligned}$$

Since the last two factors are both greater than 1, $2^p - 1$ is composite, a contradiction.

Perfect Numbers

- A **perfect number** is a positive integer that is equal to the sum of its proper positive divisors, that is the sum of its positive divisors excluding the number itself.

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- First Eight Perfect Numbers:

$$
\begin{aligned}
6 &= 1+2+3 \\
28 &= 1+2+4+7+14 \\
496 &= 1+2+4+8+16+31+62+124+248
\end{aligned}
$$

$$
8128,\ 33550336,\ 8589869056,\ 137438691328,\ 2305843008139952128
$$

Perfect Number Theorem

Theorem

An even positive integer n is perfect if and only if there exists a positive integer p such that $2^p - 1$ is prime and $n = 2^{p-1} \cdot (2^p - 1)$.

Proof

($\Leftarrow$) Let $n = 2^{p-1} \cdot (2^p - 1)$, where $2^p - 1$ is prime. Since 2^{p-1} and $2^p - 1$ are relatively prime, the sum of the divisors of n is equal to the sum of the divisors of 2^{p-1} times the sum of the divisors of $2^p - 1$. But the sum of the divisors of 2^{p-1} is

$$1 + 2 + \cdots + 2^{p-2} + 2^{p-1} = 2^p - 1$$

and the sum of the divisors of $2^p - 1$ is 2^p, since $2^p - 1$ is prime. And the product is

$$(2^p - 1) \cdot 2^p = 2 \cdot 2^{p-1}(2^p - 1) = 2n.$$

So the sum of the proper divisors of n is n and n is perfect.

Proof

($\Rightarrow$) The proof is left to the reader.

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   - Edouard Lucas
   - Computer Era
3. **48th Mersenne Prime**
   - News on 48th Mersenne Prime
4. **GIMPS**
   - GIMPS
   - GIMPS People
   - GIMPS Links
5. **Lucas-Lehmer Test and Fast Fourier Transforms**
   - Lucas-Lehmer Test
     - $2^{11} - 1$ is not prime

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4. To learn more about the distribution of Mersenne primes.

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1. To win the $150,000 offered by the Electronic Frontier Foundation (EFF) for the discovery of the first one-hundred million digit prime number. EFF's motivation is to encourage research in computational number theory related to large primes.

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Talk:
http://www.math-cs.ucmo.edu/∼curtisc/talks/gimps_cs4hs/mersennecs4hs.pdf