Mersenne Primes and GIMPS

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Prime Numbers

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- Prime Numbers Less Than 100:

$$2,\ 3,\ 5,\ 7,\ 11,\ 13,\ 17,\ 19,\ 23,\ 29,\ 31,\ 37,\ 41,$$
$$43, 47,\ 53,\ 59,\ 61,\ 67,\ 71,\ 73,\ 79,\ 83,\ 89,\ 97$$

Sieve of Eratosthenes

- One way to generate primes is by using the Sieve of Eratosthenes. An explanation of the Sieve of Eratosthenes can be found at:

  http://primes.utm.edu/glossary/page.php?sort=SieveOfEratosther

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- Demonstrations of the Sieve of Eratosthenes are found at:

  http://www.math.utah.edu/∼alfeld/Eratosthenes.html

  and

  http://www.faust.fr.bw.schule.de/mhb/eratosiv.htm

Marin Mersenne

- Mersenne primes are named after a 17th-century French monk and mathematician

Marin Mersenne (1588-1648)

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$$7 = 2^3 - 1$$
$$31 = 2^5 - 1$$
$$127 = 2^7 - 1$$
$$2047 = 2^{11} - 1$$

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- $2047 = 2^{11} - 1 = 23 \times 89.$

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- 7 divided by 3 goes 2 times with a remainder of 1. Thus, $7 \bmod 3 = 1$.
- 8 divided by 3 goes 2 times with a remainder of 2. Thus, $8 \bmod 3 = 2$.

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Examples

- 7 divided by 3 goes 2 times with a remainder of 1. Thus, $7 \bmod 3 = 1$.
- 8 divided by 3 goes 2 times with a remainder of 2. Thus, $8 \bmod 3 = 2$.
- 9 divided by 3 goes 3 times with a remainder of 0. Thus, $9 \bmod 3 = 0$.

- A Mod Calculator can be found at: http://www.antilles.k12.vi.us/math/cryptotut/mod_arithmetic.htm

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- Mod arithmetic is like clock arithmetic.

$$a \bmod b$$

  is computed by starting with a clock with b hours, from 0 to $b - 1$.
- Counting a units on the b clock, the resulting hour we end-up at is $a \bmod b$.

More Examples

- $10 \bmod 12 = 10$

More Examples

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- $14 \bmod 12 = 2$

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- $45 \bmod 11 = 1$
- $46 \bmod 7 = 4$

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- Mod Calculations are demonstrated at the website: http://www.shodor.org/interactivate/activities/clock1/

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- We can do Modular Arithmetic using the following website. http://www.math.csusb.edu/faculty/susan/modular/modcalc.html

- The **Lucas-Lehmer Test** is one way to test whether or not Mersenne numbers are Mersenne primes.

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Definition

Let $S_1 = 4$ and

$$S_{n+1} = S_n^2 - 2 \text{ for } n \geq 1.$$

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Definition

Let $S_1 = 4$ and

$$S_{n+1} = S_n^2 - 2 \text{ for } n \geq 1.$$

- The first few terms of the S sequence are:

$$4,\ 14,\ 194,\ 37634,\ 1416317954,\ 2005956546822746114,$$
$$4023861667741036022825635656102100994, \ldots$$

Lucas-Lehmer Test

Let p be a prime number. Then

$$M_p = 2^p - 1 \text{ is prime}$$
$$\text{if and only if}$$
$$S_{p-1} \bmod M_p = 0.$$

Theorem

$M_5 = 2^5 - 1 = 31$ *is prime.*

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Proof

i	S_i mod 31

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1	4
2	$(4^2 - 2) = 14 \bmod 31 = 14$

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Proof

i	$S_i \bmod 31$
1	4
2	$(4^2 - 2) = 14 \bmod 31 = 14$
3	$(14^2 - 2) = 194 \bmod 31 = 8$

Theorem

$M_5 = 2^5 - 1 = 31$ *is prime.*

Proof

i	$S_i \bmod 31$
1	4
2	$(4^2 - 2) = 14 \bmod 31 = 14$
3	$(14^2 - 2) = 194 \bmod 31 = 8$
4	$(8^2 - 2) = 62 \bmod 31 = 0$

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$M_7 = 2^7 - 1 = 127$ *is prime.*

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1	4
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Proof

i	$S_i \bmod 127$
1	4
2	$(4^2 - 2) = 14 \bmod 127 = 14$
3	$(14^2 - 2) = 194 \bmod 127 = 67$

Theorem

$M_7 = 2^7 - 1 = 127$ is prime.

Proof

i	$S_i \bmod 127$
1	4
2	$(4^2 - 2) = 14 \bmod 127 = 14$
3	$(14^2 - 2) = 194 \bmod 127 = 67$
4	$(67^2 - 2) = 4487 \bmod 127 = 42$

Theorem

$M_7 = 2^7 - 1 = 127$ is prime.

Proof

i	$S_i \bmod 127$
1	4
2	$(4^2 - 2) = 14 \bmod 127 = 14$
3	$(14^2 - 2) = 194 \bmod 127 = 67$
4	$(67^2 - 2) = 4487 \bmod 127 = 42$
5	$(42^2 - 2) = 1762 \bmod 127 = 111$

Theorem

$M_7 = 2^7 - 1 = 127$ *is prime.*

Proof

i	$S_i \bmod 127$
1	4
2	$(4^2 - 2) = 14 \bmod 127 = 14$
3	$(14^2 - 2) = 194 \bmod 127 = 67$
4	$(67^2 - 2) = 4487 \bmod 127 = 42$
5	$(42^2 - 2) = 1762 \bmod 127 = 111$
6	$(111^2 - 2) = 12319 \bmod 127 = 0$

Theorem

$M_{11} = 2^{11} - 1 = 2047$ *is not prime.*

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i	$S_i \bmod 2047$
1	4
2	$(4^2 - 2) = 14 \bmod 2047 = 14$
3	$(14^2 - 2) = 194 \bmod 2047 = 194$

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Proof

i	$S_i \bmod 2047$
1	4
2	$(4^2 - 2) = 14 \bmod 2047 = 14$
3	$(14^2 - 2) = 194 \bmod 2047 = 194$
4	$(194^2 - 2) = 37634 \bmod 2047 = 788$

Theorem

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Proof

i	$S_i \bmod 2047$
1	4
2	$(4^2 - 2) = 14 \bmod 2047 = 14$
3	$(14^2 - 2) = 194 \bmod 2047 = 194$
4	$(194^2 - 2) = 37634 \bmod 2047 = 788$
5	$(788^2 - 2) = 620942 \bmod 2047 = 701$

Proof cont.

i	S_i mod 2047

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i	$S_i \bmod 2047$
6	$(701^2 - 2) = 491399 \bmod 2047 = 119$

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7	$(119^2 - 2) = 14159 \bmod 2047 = 1877$

Proof cont.

i	$S_i \bmod 2047$
6	$(701^2 - 2) = 491399 \bmod 2047 = 119$
7	$(119^2 - 2) = 14159 \bmod 2047 = 1877$
8	$(1877^2 - 2) = 3523127 \bmod 2047 = 240$

Proof cont.

i	$S_i \bmod 2047$
6	$(701^2 - 2) = 491399 \bmod 2047 = 119$
7	$(119^2 - 2) = 14159 \bmod 2047 = 1877$
8	$(1877^2 - 2) = 3523127 \bmod 2047 = 240$
9	$(240^2 - 2) = 57598 \bmod 2047 = 282$

Proof cont.

i	$S_i \bmod 2047$
6	$(701^2 - 2) = 491399 \bmod 2047 = 119$
7	$(119^2 - 2) = 14159 \bmod 2047 = 1877$
8	$(1877^2 - 2) = 3523127 \bmod 2047 = 240$
9	$(240^2 - 2) = 57598 \bmod 2047 = 282$
10	$(282^2 - 2) = 79522 \bmod 2047 = 1736$

Theorem

$M_{31} = 2^{31} - 1 = 2147483647$ *is prime.*

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2	14

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i	$S_i \bmod 2^{31} - 1$
1	4
2	14
3	194

Theorem

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Proof

i	$S_i \bmod 2^{31} - 1$
1	4
2	14
3	194
4	37634

Theorem

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Proof

i	$S_i \bmod 2^{31} - 1$
1	4
2	14
3	194
4	37634
5	1416317954

Theorem

$M_{31} = 2^{31} - 1 = 2147483647$ *is prime.*

Proof

i	$S_i \bmod 2^{31} - 1$
1	4
2	14
3	194
4	37634
5	1416317954
6	669670838

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i	$S_i \bmod 2^{31} - 1$
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1	4
2	14
3	194
4	37634
5	1416317954
6	669670838
7	1937259419
8	425413602

Proof cont.

i	$S_i \bmod 2^{31} - 1$
9	842014276
10	12692426
11	2044502122
12	1119438707
13	1190075270
14	1450757861
15	877666528
16	630853853
17	940321271
18	512995887
19	692931217

Proof cont.

i	$S_i \bmod 2^{31} - 1$
20	1883625615
21	1992425718
22	721929267
23	27220594
24	1570086542
25	1676390412

Proof cont.

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26	1159251674
27	211987665
28	1181536708

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25	1676390412
26	1159251674
27	211987665
28	1181536708
29	65536
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Multiply two N-digit numbers A and B

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- This is called a discrete convolution.

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- The Fourier Transform allows us to do discrete convolutions very fast.
- Treat each of the numbers A and B as a vector with N components (the digits of the number).
- Do a FT on each of the two vectors to get a pair of vectors A^{FT} and B^{FT}.
- In this Fourier space, a convolution looks just like digit-by-digit multiplication.
- (That is, if we multiply each individual component (just a number) of A^{FT} with the corresponding one in B^{FT}, the result is the Fourier Transformed version of the convolution of A and B.)

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- In Fourier space, a convolution costs N operations.
- To get back the result we want, we do an inverse Fourier transform on the single vector resulting from the digit-by-digit multiply of A^{FT} and B^{FT}.

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- For N sufficiently large, this is faster than the grammar school way.
- One final thing, if we FT-multiply two numbers with N digits, we expect the product to have as many as $2N$ digits.
- Thus, we need to do FTs of length $2N$ to leave room for the digits at the high end.

FT Example - $12 \times 23 = 276$

The numbers 12 and 23 are represented as

$$A = \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \end{pmatrix} \text{ and } B = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 0 \end{pmatrix}.$$

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The numbers 12 and 23 are represented as

$$A = \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \end{pmatrix} \text{ and } B = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 0 \end{pmatrix}.$$

We use the matrix

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{pmatrix},$$

where $i = \sqrt{-1}$ to find the Fourier transform of A and B.

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We next multiply the above matrix by A and B to find their Fourier transforms.

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We next multiply the above matrix by A and B to find their Fourier transforms. Therefore,

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And

$$B^{FT} = \begin{pmatrix} 5 \\ 3+2i \\ 1 \\ 3-2i \end{pmatrix}.$$

FT Discrete Convolution of A and B

Doing component-by-component multiplication, $A^{FT} * B^{FT}$ gives

$$\begin{pmatrix} 3 \\ 2+i \\ 1 \\ 2-i \end{pmatrix} * \begin{pmatrix} 5 \\ 3+2i \\ 1 \\ 3-2i \end{pmatrix} = \begin{pmatrix} 15 \\ 4+7i \\ 1 \\ 4-7i \end{pmatrix}.$$

Inverse Fourier Transform of Vector

The inverse Fourier transform of this vector uses the Fourier transform matrix, but the signs on the i-terms are switched and a factor of $1/4$ multiplying the whole matrix.

$$\frac{1}{4}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{pmatrix}.$$

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$$\frac{1}{4}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{pmatrix}.$$

Multiplying this matrix by our vector gives

$$\begin{pmatrix} 6 \\ 7 \\ 2 \\ 0 \end{pmatrix}.$$

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exponent	Digits in M_p	year	discoverer

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	exponent	Digits in M_p	year	discoverer
1	2	1	—	—
2	3	1	—	—
3	5	2	—	—
4	7	3	—	—

Before Computers

	exponent	Digits in M_p	year	discoverer
1	2	1	—	—
2	3	1	—	—
3	5	2	—	—
4	7	3	—	—
5	13	4	1456	anonymous

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4	7	3	—	—
5	13	4	1456	anonymous
6	17	6	1588	Cataldi
7	19	6	1588	Cataldi

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8	31	10	1772	Euler

9	61	19	1883	Pervushin
10	89	27	1911	Powers
11	107	33	1914	Powers

Before Computers

	exponent	Digits in M_p	year	discoverer
12	127	39	1876	Lucas

Mainframe and Supercomputer Era

	exponent	Digits in M_p	year	discoverer
13	521	157	1952	Robinson
14	607	183	1952	Robinson
15	1279	386	1952	Robinson
16	2203	664	1952	Robinson
17	2281	687	1952	Robinson

Mainframe and Supercomputer Era

	exponent	Digits in M_p	year	discoverer
18	3217	969	1957	Riesel

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22	9941	2993	1963	Gillies
23	11213	3376	1963	Gillies

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23	11213	3376	1963	Gillies
24	19937	6002	1971	Tuckerman

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25	21701	6533	1978	Noll and Nickel
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28	86243	25962	1982	Slowinski
29	110503	33265	1988	Colquitt and Welsh

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30	132049	39751	1983	Slowinski
31	216091	65050	1985	Slowinski

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30	132049	39751	1983	Slowinski
31	216091	65050	1985	Slowinski
32	756839	227832	1992	Slowinski and Gage
33	859433	258716	1994	Slowinski and Gage
34	1257787	378632	1996	Slowinski and Gage

GIMPS (Woltman, Kurowski, et al.) Era

	exponent	Digits in M_p	year	discoverer
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37	3021377	909526	1998	Clarkson, GIMPS
38	6972593	2098960	1999	Hajratwala, GIMPS

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39?	13466917	4053946	2001	Cameron, GIMPS

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39?	13466917	4053946	2001	Cameron, GIMPS
40?	20996011	6320430	2003	Shafer, GIMPS

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41?	24036583	7235733	2004	Findley, GIMPS

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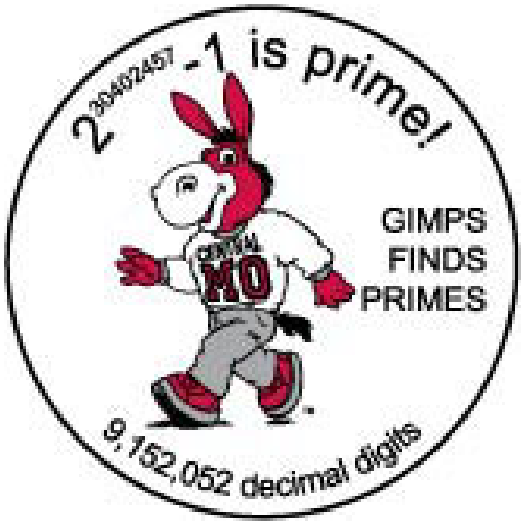
	exponent	Digits in M_p	year	discoverer
41?	24036583	7235733	2004	Findley, GIMPS
42?	25964951	7816230	2005	Nowak, GIMPS
43?	30402457	9152052	2005	Cooper, Boone, CMSU, GIMPS

$2^{30402457} - 1$ Button



News About $2^{30402457} - 1$

- On December 15, 2005 at 8:46:58 am (CST), computer commwd102–07l in the Communications Lab (Wood 102) proved that $2^{30402457} - 1$ is prime.

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- News items on the web regarding M30402457 can be found at: http://www.math-cs.cmsu.edu/∼curtisc/M30402457.html

Digits of $2^{30402457} - 1$

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- Scott Kurowski wrote the PrimeNet server that supports GIMPS. In 1997 he founded Entropia, a distributed computing software company.

GIMPS

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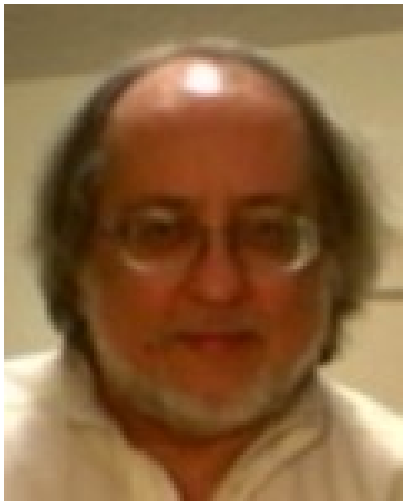
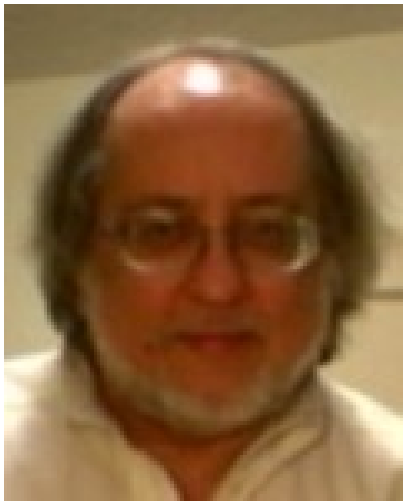
- The GIMPS project consists of 70,000 networked computers.
- CMSU has over 850 computers performing LL-tests on Mersenne numbers.

GIMPS People



GIMPS People

Woltman



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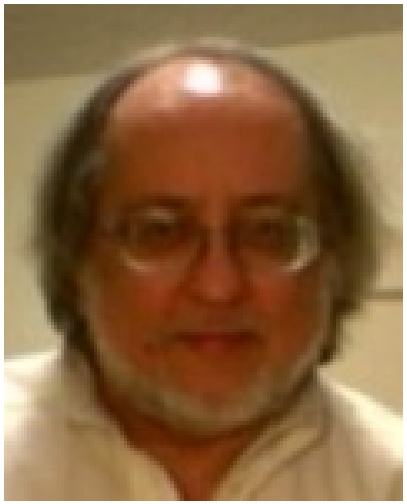

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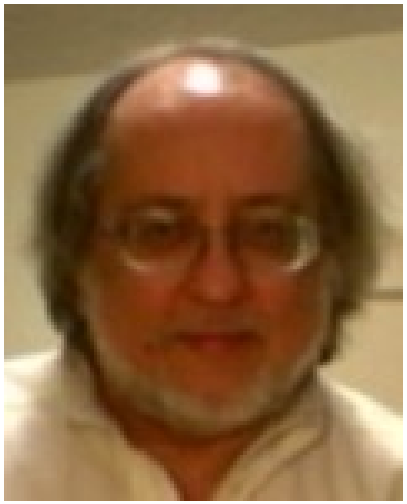

Crandall

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- Jeff Gilchrist of Elytra Enterprises Inc. in Ottawa, Canada, using fourteen days of time on 14 CPUs of a Compaq Alpha GS160 1.2 GHz CPU server at SHARCNET, triple-checked M30402457.

GIMPS People

GIMPS People

T. Rex

GIMPS People

T. Rex

Valor

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Gilchrist

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Top 10

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7. To discover new and more efficient algorithms for testing the primality of large numbers.

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6. To help detect hardware problems (fan and CPU/bus problems) on individual computers at CMSU.

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4. To learn more about the distribution of Mersenne primes.

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1. To win the $100,000 offered by the Electronic Frontier Foundation (EFF) for the discovery of the first ten million digit prime number. EFF's motivation is to encourage research in computational number theory related to large primes.