

# The 48th Mersenne Prime, GIMPS, the LL Test, and Perfect Numbers

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# 1 Mersenne Primes

- Primes
- Mersenne Primes

## 2 History of Mersenne Primes

- Marin Mersenne
- Edouard Lucas
- Computer Era

## 3 48th Mersenne Prime

- News on 48th Mersenne Prime

## 4 GIMPS

- GIMPS
- GIMPS People
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## 5 Lucas-Lehmer Test and Fast Fourier Transforms

- Lucas-Lehmer Test
  - $2^{11} - 1$  is not prime

# Prime Numbers

- A **prime number** is an integer, greater than 1, which has exactly two factors, itself and one.

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- A **prime number** is an integer, greater than 1, which has exactly two factors, itself and one.
- Prime Numbers Less Than 100:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,  
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

# Mersenne Numbers

- A **Mersenne number** is a number of the form  $2^p - 1$ , where  $p$  is a prime number.

# Mersenne Numbers

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- Examples of Mersenne numbers are:

$$M_2 = 3 = 2^2 - 1$$

$$M_3 = 7 = 2^3 - 1$$

$$M_5 = 31 = 2^5 - 1$$

$$M_7 = 127 = 2^7 - 1$$

$$M_{11} = 2047 = 2^{11} - 1$$

# Mersenne Primes

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# Mersenne Primes

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$$7 = 2^3 - 1$$

$$31 = 2^5 - 1$$

$$127 = 2^7 - 1$$

$$8191 = 2^{13} - 1$$

- $2047 = 2^{11} - 1 = 23 \times 89$ .

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# Marin Mersenne

- Mersenne primes are named after a 17th-century French monk and mathematician



Marin Mersenne (1588-1648)

- Mersenne compiled what was supposed to be a list of Mersenne primes with exponents up to 257.

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- His list was largely incorrect, as Mersenne mistakenly included M67 and M257 (which are composite), and omitted M61, M89, and M107 (which are prime).

## Edouard Lucas



Edouard Lucas

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Edouard Lucas

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- Lucas proved in 1876 that  $M_{127}$  is indeed prime, as Mersenne claimed. This was the largest known prime number for 75 years, and the largest ever calculated by hand.
- Without finding a factor, Lucas demonstrated that  $M_{67}$  is actually composite.



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- On the other side of the board, he multiplied 193,707,721 times 761,838,257,287 and got the same number, then returned to his seat (to applause) without speaking.

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- Without speaking a word, he went to a blackboard and raised 2 to the 67th power, then subtracted one.
- On the other side of the board, he multiplied 193,707,721 times 761,838,257,287 and got the same number, then returned to his seat (to applause) without speaking.
- A correct list of all Mersenne primes in this number range was completed and rigorously verified only about three centuries after Mersenne published his list.

## Computer Era

- The search for Mersenne primes was revolutionized by the introduction of the electronic digital computer.

## Computer Era

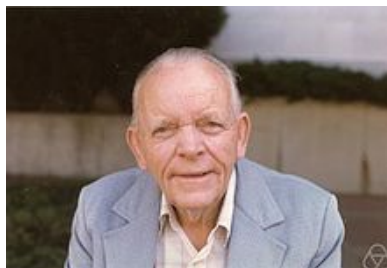
- The search for Mersenne primes was revolutionized by the introduction of the electronic digital computer.
- Landon Curt Noll and Laura Nickel, 18 year-old high school students, discovered M21701. They were both studying number theory under Dr. Lehmer. This is the 25th Mersenne prime.

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- Later Landon Curt Noll found M23209.

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Derrick Henry Lehmer



## Computer Era

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- M6,972,593 was the first prime with at least 1,000,000 digits.

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- M44497 is the first with more than 10,000 digits.
- M6,972,593 was the first prime with at least 1,000,000 digits.
- All three were the first known prime of any kind of that size.

## Computer Era

- In September 2008, Edson Smith at UCLA, participating in GIMPS, won part of a 100,000 dollar prize from the Electronic Frontier Foundation for their discovery of a very nearly 13-million-digit Mersenne prime.

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- The prize, finally confirmed in October 2009, is for the first known prime with at least 10 million digits.
- The prime was found on a Dell OptiPlex 745 on August 23, 2008. This is the eighth Mersenne prime discovered at UCLA.

- List of 48 Known Mersenne Primes -  
[http://en.wikipedia.org/wiki/Mersenne\\_prime](http://en.wikipedia.org/wiki/Mersenne_prime)



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# News About 48th Mersenne Prime

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# More About 48th Mersenne Prime

- Fox 4 Kansas City News Story  
<http://fox4kc.com/2013/02/08/ucm-professors-big-prime-number-discovery-has-bragging-rights/>

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- Lee Judge Cartoon  
<http://www.cartoonistgroup.com/subject/The-Judge-Comics-and-Cartoons.php>

# Mersenne Buttons

- M30402457 Button <http://www.math-cs.ucmo.edu/~curtisc/photos/M30402457.jpg>

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- M30402457 Button <http://www.math-cs.ucmo.edu/~curtisc/photos/M30402457.jpg>
- M32582657 Button <http://www.math-cs.ucmo.edu/~curtisc/photos/M32582657.jpg>



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- M30402457 Button <http://www.math-cs.ucmo.edu/~curtisc/photos/M30402457.jpg>
- M32582657 Button <http://www.math-cs.ucmo.edu/~curtisc/photos/M32582657.jpg>
- M57885161 Button  
<http://www.math-cs.ucmo.edu/~curtisc/images/1.jpg>

# Jumping GIFS

- 3 Primes GIF

<http://www.math-cs.ucmo.edu/~curtisc/images/6.gif>

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<http://www.math-cs.ucmo.edu/~curtisc/images/6.gif>

- UCM GIF

<http://www.math-cs.ucmo.edu/~curtisc/images/14.gif>

# Digits of M57885161

- Digits of M57885161  
<http://www.isthe.com/chongo/tech/math/digit/m57885161/prime-c.html>

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<http://www.isthe.com/chongo/tech/math/digit/m57885161/prime-d.html>

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- GIMPS is a collaborative project of volunteers who are searching for Mersenne prime numbers. The software used by GIMPS volunteers is Prime95. This software can be downloaded from the Internet for free.

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- George Woltman founded GIMPS in January 1996 and wrote the prime testing software.



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- George Woltman founded GIMPS in January 1996 and wrote the prime testing software.
- Scott Kurowski wrote the PrimeNet server that supports GIMPS. In 1997 he founded Entropia, a distributed computing software company.

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- As of February 5, 2013, GIMPS had a sustained throughput of approximately 129 trillion floating-point operations per second).
- The GIMPS project consists of 98,980 users, 574 teams, and 730,562 CPUs.
- UCM has over 1000 computers performing LL-tests on Mersenne numbers.

## GIMPS People



Woltman



Kurowski



Crandall

- The GIMPS home page can be found at:  
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- A Mersenne Prime discussion forum can be found at:  
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## Lucas-Lehmer Test

- The **Lucas-Lehmer Test** is one way to test whether or not Mersenne numbers are Mersenne primes.

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### Definition

Let  $S_1 = 4$  and

$$S_{n+1} = S_n^2 - 2 \text{ for } n \geq 1.$$

- The **Lucas-Lehmer Test** is one way to test whether or not Mersenne numbers are Mersenne primes.

### Definition

Let  $S_1 = 4$  and

$$S_{n+1} = S_n^2 - 2 \text{ for } n \geq 1.$$

- The first few terms of the  $S$  sequence are:

4, 14, 194, 37634, 1416317954, 2005956546822746114,  
4023861667741036022825635656102100994, ...

## Lucas-Lehmer Test

Let  $p$  be a prime number. Then

$M_p = 2^p - 1$  is prime

if and only if

$$S_{p-1} \bmod M_p = 0.$$

## Lucas-Lehmer Test



Lucas



Lehmer

## Theorem

$M_{11} = 2^{11} - 1 = 2047$  is not prime.

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## Proof

$i$

$S_i \bmod 2047$



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|-----|------------------|
| 1   | 4                |

## Theorem

$M_{11} = 2^{11} - 1 = 2047$  is not prime.

## Proof

| $i$ | $S_i \bmod 2047$                 |
|-----|----------------------------------|
| 1   | 4                                |
| 2   | $(4^2 - 2) = 14 \bmod 2047 = 14$ |

## Theorem

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## Proof

| $i$ | $S_i \bmod 2047$                    |
|-----|-------------------------------------|
| 1   | 4                                   |
| 2   | $(4^2 - 2) = 14 \bmod 2047 = 14$    |
| 3   | $(14^2 - 2) = 194 \bmod 2047 = 194$ |

## Theorem

$M_{11} = 2^{11} - 1 = 2047$  is not prime.

## Proof

| $i$ | $S_i \bmod 2047$                       |
|-----|--|
| 1   | 4                                      |
| 2   | $(4^2 - 2) = 14 \bmod 2047 = 14$       |
| 3   | $(14^2 - 2) = 194 \bmod 2047 = 194$    |
| 4   | $(194^2 - 2) = 37634 \bmod 2047 = 788$ |

## Theorem

$M_{11} = 2^{11} - 1 = 2047$  is not prime.

## Proof

| $i$ | $S_i \bmod 2047$                        |
|-----|---|
| 1   | 4                                       |
| 2   | $(4^2 - 2) = 14 \bmod 2047 = 14$        |
| 3   | $(14^2 - 2) = 194 \bmod 2047 = 194$     |
| 4   | $(194^2 - 2) = 37634 \bmod 2047 = 788$  |
| 5   | $(788^2 - 2) = 620942 \bmod 2047 = 701$ |

## Lucas-Lehmer Test

# $2^{11} - 1$ is not prime

## Proof cont.

 $i$  $S_i \bmod 2047$

## Lucas-Lehmer Test

 $2^{11} - 1$  is not prime

## Proof cont.

|     |   |
|-----|---|
| $i$ | $S_i \bmod 2047$                        |
| 6   | $(701^2 - 2) = 491399 \bmod 2047 = 119$ |

## Lucas-Lehmer Test

# $2^{11} - 1$ is not prime

**Proof cont.**

| $i$ | $S_i \bmod 2047$                        |
|-----|---|
| 6   | $(701^2 - 2) = 491399 \bmod 2047 = 119$ |
| 7   | $(119^2 - 2) = 14159 \bmod 2047 = 1877$ |



# $2^{11} - 1$ is not prime

## Proof cont.

| $i$ | $S_i \bmod 2047$                          |
|-----|---|
| 6   | $(701^2 - 2) = 491399 \bmod 2047 = 119$   |
| 7   | $(119^2 - 2) = 14159 \bmod 2047 = 1877$   |
| 8   | $(1877^2 - 2) = 3523127 \bmod 2047 = 240$ |

# $2^{11} - 1$ is not prime

## Proof cont.

| $i$ | $S_i \bmod 2047$                          |
|-----|---|
| 6   | $(701^2 - 2) = 491399 \bmod 2047 = 119$   |
| 7   | $(119^2 - 2) = 14159 \bmod 2047 = 1877$   |
| 8   | $(1877^2 - 2) = 3523127 \bmod 2047 = 240$ |
| 9   | $(240^2 - 2) = 57598 \bmod 2047 = 282$    |

# $2^{11} - 1$ is not prime

## Proof cont.

| $i$ | $S_i \bmod 2047$                          |
|-----|---|
| 6   | $(701^2 - 2) = 491399 \bmod 2047 = 119$   |
| 7   | $(119^2 - 2) = 14159 \bmod 2047 = 1877$   |
| 8   | $(1877^2 - 2) = 3523127 \bmod 2047 = 240$ |
| 9   | $(240^2 - 2) = 57598 \bmod 2047 = 282$    |
| 10  | $(282^2 - 2) = 79522 \bmod 2047 = 1736$   |

## Theorem

$M_{31} = 2^{31} - 1 = 2147483647$  is prime.

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Proof.

|     |                        |
|-----|------------------------|
| $i$ | $S_i \bmod 2^{31} - 1$ |
| 1   | 4                      |

**Theorem**

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Proof.

| $i$ | $S_i \bmod 2^{31} - 1$ |
|-----|------------------------|
| 1   | 4                      |
| 2   | 14                     |

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Proof.

| $i$ | $S_i \bmod 2^{31} - 1$ |
|-----|------------------------|
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| 2   | 14                     |
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## Theorem

$M_{31} = 2^{31} - 1 = 2147483647$  is prime.

Proof.

| $i$ | $S_i \bmod 2^{31} - 1$ |
|-----|------------------------|
| 1   | 4                      |
| 2   | 14                     |
| 3   | 194                    |
| 4   | 37634                  |

## Theorem

$M_{31} = 2^{31} - 1 = 2147483647$  is prime.

Proof.

| $i$ | $S_i \bmod 2^{31} - 1$ |
|-----|------------------------|
| 1   | 4                      |
| 2   | 14                     |
| 3   | 194                    |
| 4   | 37634                  |
| 5   | 1416317954             |

## Theorem

$M_{31} = 2^{31} - 1 = 2147483647$  is prime.

Proof.

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|-----|------------------------|
| 1   | 4                      |
| 2   | 14                     |
| 3   | 194                    |
| 4   | 37634                  |
| 5   | 1416317954             |
| 6   | 669670838              |

## Theorem

$M_{31} = 2^{31} - 1 = 2147483647$  is prime.

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| $i$ | $S_i \bmod 2^{31} - 1$ |
|-----|------------------------|
| 1   | 4                      |
| 2   | 14                     |
| 3   | 194                    |
| 4   | 37634                  |
| 5   | 1416317954             |
| 6   | 669670838              |
| 7   | 1937259419             |

## Theorem

$M_{31} = 2^{31} - 1 = 2147483647$  is prime.

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| $i$ | $S_i \bmod 2^{31} - 1$ |
|-----|------------------------|
| 1   | 4                      |
| 2   | 14                     |
| 3   | 194                    |
| 4   | 37634                  |
| 5   | 1416317954             |
| 6   | 669670838              |
| 7   | 1937259419             |
| 8   | 425413602              |

## Lucas-Lehmer Test

 $2^{31} - 1$  is prime

| $i$ | $S_i \bmod 2^{31} - 1$ |
|-----|------------------------|
| 9   | 842014276              |
| 10  | 12692426               |
| 11  | 2044502122             |
| 12  | 1119438707             |
| 13  | 1190075270             |
| 14  | 1450757861             |
| 15  | 877666528              |
| 16  | 630853853              |
| 17  | 940321271              |
| 18  | 512995887              |
| 19  | 692931217              |

## Lucas-Lehmer Test

 $2^{31} - 1$  is prime

| $i$ | $S_i \bmod 2^{31} - 1$ |
|-----|------------------------|
| 20  | 1883625615             |
| 21  | 1992425718             |
| 22  | 721929267              |
| 23  | 27220594               |
| 24  | 1570086542             |
| 25  | 1676390412             |

## Lucas-Lehmer Test

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|-----|------------------------|
| 20  | 1883625615             |
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| 26  | 1159251674             |



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| $i$ | $S_i \bmod 2^{31} - 1$ |
|-----|------------------------|
| 20  | 1883625615             |
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| 22  | 721929267              |
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| 26  | 1159251674             |
| 27  | 211987665              |

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 $2^{31} - 1$  is prime

| $i$ | $S_i \bmod 2^{31} - 1$ |
|-----|------------------------|
| 20  | 1883625615             |
| 21  | 1992425718             |
| 22  | 721929267              |
| 23  | 27220594               |
| 24  | 1570086542             |
| 25  | 1676390412             |
| 26  | 1159251674             |
| 27  | 211987665              |
| 28  | 1181536708             |

## Lucas-Lehmer Test

 $2^{31} - 1$  is prime

| $i$ | $S_i \bmod 2^{31} - 1$ |
|-----|------------------------|
| 20  | 1883625615             |
| 21  | 1992425718             |
| 22  | 721929267              |
| 23  | 27220594               |
| 24  | 1570086542             |
| 25  | 1676390412             |
| 26  | 1159251674             |
| 27  | 211987665              |
| 28  | 1181536708             |
| 29  | 65536                  |

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 $2^{31} - 1$  is prime

| $i$ | $S_i \bmod 2^{31} - 1$ |
|-----|------------------------|
| 20  | 1883625615             |
| 21  | 1992425718             |
| 22  | 721929267              |
| 23  | 27220594               |
| 24  | 1570086542             |
| 25  | 1676390412             |
| 26  | 1159251674             |
| 27  | 211987665              |
| 28  | 1181536708             |
| 29  | 65536                  |
| 30  | 0                      |

# Fast Fourier Transforms

- Fast Fourier Transform Paper  
<http://www.math-cs.ucmo.edu/~curtisc/M57885161.html>

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# 5 Lucas-Lehmer Test and Fast Fourier Transforms

- Lucas-Lehmer Test

●  $2^{11} - 1$  is not prime

## Theorem

*If  $p$  is an odd prime, then any prime  $q$  that divides  $2^p - 1$  must be 1 plus a multiple of  $2p$ . This holds even when  $2^p - 1$  is prime*



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- $2^5 - 1 = 31$  is prime and  $31 = 1 + 3 \times 2 \times 5$ .
- $2^{11} - 1 = 2047 = 23 \times 89$ , where  $23 = 1 + 2 \times 11$  and  $89 = 1 + 4 \times 2 \times 11$ .

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- Primes 47 and 178481 divide  $2^{23} - 1 = 8,388,607$ .  $47 \equiv -1 \pmod{8}$  and  $178481 \equiv 1 \pmod{8}$ .

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## Proof

By contradiction. Suppose  $p$  is composite. Then  $p = ab$  for some  $a, b > 1$ . But then

$$\begin{aligned} 2^p - 1 &= 2^{ab} - 1 = (2^a)^b - 1 \\ &= (2^a - 1) \cdot (2^{a(b-1)} + 2^{a(b-2)} + \dots + 2^a + 1). \end{aligned}$$

Since the last two factors are both greater than 1,  $2^p - 1$  is composite, a contradiction.

# Perfect Numbers

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- First Eight Perfect Numbers:

$$6 = 1 + 2 + 3$$

$$28 = 1 + 2 + 4 + 7 + 14$$

$$496 = 1 + 2 + 4 + 8 + 16 + 31 + 62 + 124 + 248$$

$$8128, 33550336, 8589869056,$$

$$137438691328, 2305843008139952128$$



# Perfect Number Theorem

## Theorem

*An even positive integer  $n$  is perfect if and only if there exists a positive integer  $p$  such that  $2^p - 1$  is prime and  $n = 2^{p-1} \cdot (2^p - 1)$ .*



## Proof

( $\Leftarrow$ ) Let  $n = 2^{p-1} \cdot (2^p - 1)$ , where  $2^p - 1$  is prime. Since  $2^{p-1}$  and  $2^p - 1$  are relatively prime, the sum of the divisors of  $n$  is equal to the sum of the divisors of  $2^{p-1}$  times the sum of the divisors of  $2^p - 1$ . But the sum of the divisors of  $2^{p-1}$  is

$$1 + 2 + \dots + 2^{p-2} + 2^{p-1} = 2^p - 1$$

and the sum of the divisors of  $2^p - 1$  is  $2^p$ , since  $2^p - 1$  is prime. And the product is

$$(2^p - 1) \cdot 2^p = 2 \cdot 2^{p-1} (2^p - 1) = 2n.$$

So the sum of the proper divisors of  $n$  is  $n$  and  $n$  is perfect.

## Proof

( $\Rightarrow$ ) The proof is left to the reader.

# 1 Mersenne Primes

- Primes
- Mersenne Primes

# 2 History of Mersenne Primes

- Marin Mersenne
- Edouard Lucas
- Computer Era

# 3 48th Mersenne Prime

- News on 48th Mersenne Prime

# 4 GIMPS

- GIMPS
- GIMPS People
- GIMPS Links

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4. To learn more about the distribution of Mersenne primes.

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2. To produce much favorable press for UCM and demonstrate that the University of Central Missouri is a first-class research and teaching institution.
1. To win the \$150,000 offered by the Electronic Frontier Foundation (EFF) for the discovery of the first one-hundred million digit prime number. EFF's motivation is to encourage research in computational number theory related to large primes.

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