The 48th Mersenne Prime, GIMPS, and the LL Test and FFTs

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University of Central Missouri

February 18, 2013

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- News on 48th Mersenne Prime

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  - $2^{11} - 1$ is not prime
  - $2^{31} - 1$ is prime
- Fast Fourier Transforms

Prime Numbers

- A **prime number** is an integer, greater than 1, which has exactly two factors, itself and one.

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- Prime Numbers Less Than 100:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

Mersenne Numbers

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- Examples of Mersenne numbers are:

$$3 = 2^2 - 1$$
$$7 = 2^3 - 1$$
$$31 = 2^5 - 1$$
$$127 = 2^7 - 1$$
$$2047 = 2^{11} - 1$$

Mersenne Primes

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- $2047 = 2^{11} - 1 = 23 \times 89$.

Marin Mersenne

- Mersenne primes are named after a 17th-century French monk and mathematician

Marin Mersenne (1588-1648)

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- Pronunciation of M57885161
  http://www.isthe.com/chongo/tech/math/digit/m57885161/prime-d.html

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The Great Internet Mersenne Prime Search

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- George Woltman founded GIMPS in January 1996 and wrote the prime testing software.
- Scott Kurowski wrote the PrimeNet server that supports GIMPS. In 1997 he founded Entropia, a distributed computing software company.

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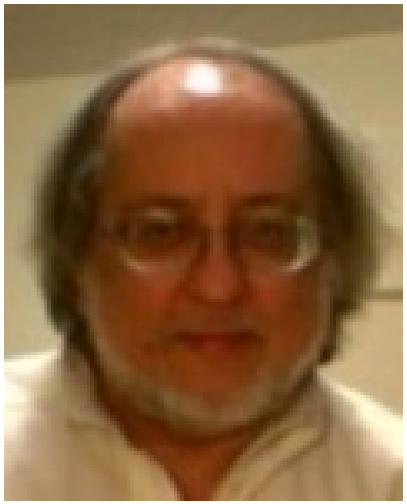
- The GIMPS project consists of 88,074 users, 539 teams, and 642,683 CPUs.
- UCM has over 1000 computers performing LL-tests on Mersenne numbers.

GIMPS People

Woltman

Kurowski



Crandall

- The GIMPS home page can be found at: http://www.mersenne.org

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- A Mersenne Prime discussion forum can be found at: http://www.mersenneforum.org

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Definition

Let $S_1 = 4$ and

$$S_{n+1} = S_n^2 - 2 \text{ for } n \geq 1.$$

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Definition

Let $S_1 = 4$ and

$$S_{n+1} = S_n^2 - 2 \text{ for } n \geq 1.$$

- The first few terms of the S sequence are:

$$4,\ 14,\ 194,\ 37634,\ 1416317954,\ 2005956546822746114,$$
$$4023861667741036022825635656102100994, \ldots$$

Lucas-Lehmer Test

Let p be a prime number. Then

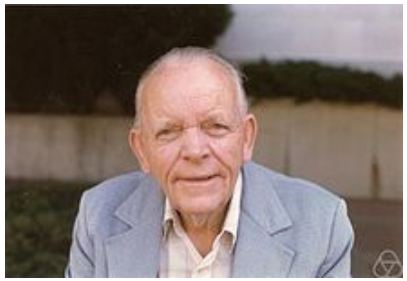
$$M_p = 2^p - 1 \text{ is prime}$$

if and only if

$$S_{p-1} \bmod M_p = 0.$$

Lucas-Lehmer Test

Lucas



Lehmer

Theorem

$M_{11} = 2^{11} - 1 = 2047$ *is not prime.*

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Proof

i	$S_i \bmod 2047$

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Proof

i	$S_i \bmod 2047$
1	4

Theorem

$M_{11} = 2^{11} - 1 = 2047$ *is not prime.*

Proof

i	$S_i \bmod 2047$
1	4
2	$(4^2 - 2) = 14 \bmod 2047 = 14$

Theorem

$M_{11} = 2^{11} - 1 = 2047$ *is not prime.*

Proof

i	$S_i \bmod 2047$
1	4
2	$(4^2 - 2) = 14 \bmod 2047 = 14$
3	$(14^2 - 2) = 194 \bmod 2047 = 194$

Theorem

$M_{11} = 2^{11} - 1 = 2047$ *is not prime.*

Proof

i	$S_i \bmod 2047$
1	4
2	$(4^2 - 2) = 14 \bmod 2047 = 14$
3	$(14^2 - 2) = 194 \bmod 2047 = 194$
4	$(194^2 - 2) = 37634 \bmod 2047 = 788$

Theorem

$M_{11} = 2^{11} - 1 = 2047$ *is not prime.*

Proof

i	$S_i \bmod 2047$
1	4
2	$(4^2 - 2) = 14 \bmod 2047 = 14$
3	$(14^2 - 2) = 194 \bmod 2047 = 194$
4	$(194^2 - 2) = 37634 \bmod 2047 = 788$
5	$(788^2 - 2) = 620942 \bmod 2047 = 701$

$2^{11} - 1$ is not prime

Proof cont.

i	$S_i \bmod 2047$

$2^{11} - 1$ is not prime

Proof cont.

i	$S_i \bmod 2047$
6	$(701^2 - 2) = 491399 \bmod 2047 = 119$

$2^{11} - 1$ is not prime

Proof cont.

i	$S_i \bmod 2047$
6	$(701^2 - 2) = 491399 \bmod 2047 = 119$
7	$(119^2 - 2) = 14159 \bmod 2047 = 1877$

$2^{11} - 1$ is not prime

Proof cont.

i	$S_i \bmod 2047$
6	$(701^2 - 2) = 491399 \bmod 2047 = 119$
7	$(119^2 - 2) = 14159 \bmod 2047 = 1877$
8	$(1877^2 - 2) = 3523127 \bmod 2047 = 240$

$2^{11} - 1$ is not prime

Proof cont.

i	$S_i \bmod 2047$
6	$(701^2 - 2) = 491399 \bmod 2047 = 119$
7	$(119^2 - 2) = 14159 \bmod 2047 = 1877$
8	$(1877^2 - 2) = 3523127 \bmod 2047 = 240$
9	$(240^2 - 2) = 57598 \bmod 2047 = 282$

$2^{11} - 1$ is not prime

Proof cont.

i	$S_i \bmod 2047$
6	$(701^2 - 2) = 491399 \bmod 2047 = 119$
7	$(119^2 - 2) = 14159 \bmod 2047 = 1877$
8	$(1877^2 - 2) = 3523127 \bmod 2047 = 240$
9	$(240^2 - 2) = 57598 \bmod 2047 = 282$
10	$(282^2 - 2) = 79522 \bmod 2047 = 1736$

Theorem

$M_{31} = 2^{31} - 1 = 2147483647$ *is prime.*

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Proof.

i	$S_i \bmod 2^{31} - 1$

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Proof.

i	$S_i \bmod 2^{31} - 1$
1	4

Theorem

$M_{31} = 2^{31} - 1 = 2147483647$ *is prime.*

Proof.

i	$S_i \bmod 2^{31} - 1$
1	4
2	14

Theorem

$M_{31} = 2^{31} - 1 = 2147483647$ *is prime.*

Proof.

i	$S_i \bmod 2^{31} - 1$
1	4
2	14
3	194

Theorem

$M_{31} = 2^{31} - 1 = 2147483647$ *is prime.*

Proof.

i	$S_i \bmod 2^{31} - 1$
1	4
2	14
3	194
4	37634

Theorem

$M_{31} = 2^{31} - 1 = 2147483647$ *is prime.*

Proof.

i	$S_i \bmod 2^{31} - 1$
1	4
2	14
3	194
4	37634
5	1416317954

Theorem

$M_{31} = 2^{31} - 1 = 2147483647$ *is prime.*

Proof.

i	$S_i \bmod 2^{31} - 1$
1	4
2	14
3	194
4	37634
5	1416317954
6	669670838

Theorem

$M_{31} = 2^{31} - 1 = 2147483647$ *is prime.*

Proof.

i	$S_i \bmod 2^{31} - 1$
1	4
2	14
3	194
4	37634
5	1416317954
6	669670838
7	1937259419

Theorem

$M_{31} = 2^{31} - 1 = 2147483647$ *is prime.*

Proof.

i	$S_i \bmod 2^{31} - 1$
1	4
2	14
3	194
4	37634
5	1416317954
6	669670838
7	1937259419
8	425413602

$2^{31} - 1$ is prime

i	$S_i \bmod 2^{31} - 1$
9	842014276
10	12692426
11	2044502122
12	1119438707
13	1190075270
14	1450757861
15	877666528
16	630853853
17	940321271
18	512995887
19	692931217

$2^{31} - 1$ is prime

i	$S_i \bmod 2^{31} - 1$
20	1883625615
21	1992425718
22	721929267
23	27220594
24	1570086542
25	1676390412

$2^{31} - 1$ is prime

i	$S_i \bmod 2^{31} - 1$
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26	1159251674

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26	1159251674
27	211987665

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21	1992425718
22	721929267
23	27220594
24	1570086542
25	1676390412
26	1159251674
27	211987665
28	1181536708

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26	1159251674
27	211987665
28	1181536708
29	65536

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23	27220594
24	1570086542
25	1676390412
26	1159251674
27	211987665
28	1181536708
29	65536
30	0

Fast Fourier Transforms

- Fast Fourier Transform Paper
  http://www.math-cs.ucmo.edu/ curtisc/M57885161.html

Email Address and Talk URL

Curtis Cooper's Email:
cooper@ucmo.edu

Talk:
http://www.math-cs.ucmo.edu/∼curtisc/talks/gimpsacm/mersenneacm.pdf