Mersenne Primes, GIMPS, and the LL Test

Curtis Cooper
University of Central Missouri

June 13, 2018

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   - Mersenne Primes
2. **History of Mersenne Primes**
   - Marin Mersenne
   - Edouard Lucas
   - Computer Era
3. **50th Mersenne Prime**
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   - News on 50th Mersenne Prime
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Prime Numbers

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Prime Numbers

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- Prime Numbers Less Than 100:

$$2,\ 3,\ 5,\ 7,\ 11,\ 13,\ 17,\ 19,\ 23,\ 29,\ 31,\ 37,\ 41,$$
$$43,\ 47,\ 53,\ 59,\ 61,\ 67,\ 71,\ 73,\ 79,\ 83,\ 89,\ 97$$

Mersenne Numbers

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- Examples of Mersenne numbers are:

$$M2 = 2^2 - 1 = 3$$
$$M3 = 2^3 - 1 = 7$$
$$M5 = 2^5 - 1 = 31$$
$$M7 = 2^7 - 1 = 127$$
$$M11 = 2^{11} - 1 = 2047$$

Mersenne Primes

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$$
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7 &= 2^3 - 1 \\
31 &= 2^5 - 1 \\
127 &= 2^7 - 1 \\
8191 &= 2^{13} - 1
\end{aligned}
$$

- $2047 = 2^{11} - 1 = 23 \times 89.$

Marin Mersenne

- Mersenne primes are named after a 17th-century French monk and mathematician

Marin Mersenne (1588-1648)

Marin Mersenne

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- 2, 3, 5, 7, 13, 17, 19, 31, 67, 127, 257

Marin Mersenne

- Mersenne compiled what was supposed to be a list of Mersenne primes with exponents up to 257.
- 2, 3, 5, 7, 13, 17, 19, 31, 67, 127, 257
- His list was largely incorrect, as Mersenne mistakenly included M67 and M257 (which are composite), and omitted M61, M89, and M107 (which are prime).

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- Without finding a factor, Lucas demonstrated that M67 is actually composite.

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Computer Era

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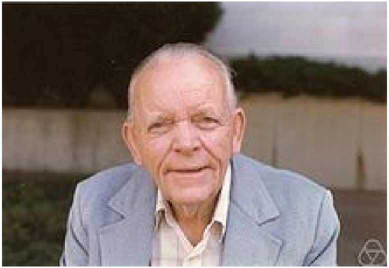

Derrick Henry Lehmer

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- The prime was found on a Dell OptiPlex 745. This is the eighth Mersenne prime discovered at UCLA.

- List of 50 Known Mersenne Primes - https://en.wikipedia.org/wiki/Mersenne_prime

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- As SysAdmin for his charities, he runs Prime95 on all PCs and servers.

News About 50th Mersenne Prime

- Official Press Release https://www.mersenne.org/primes/press/M77232917.html

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- Science Daily
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- John D. Cook
  https://www.johndcook.com/blog/2018/01/04/new-prime-number-record-50th-mersenne-prime/

Digits of M77232917 by Landon Curt Noll

- Digits of M77232917 http://lcn2.github.io/mersenne-english-name/m77232917/prime-c.html

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UCM's Four Mersenne Primes

- M30402457
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More About 49th Mersenne Prime

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  https://www.facebook.com/kshbtv/videos/10153315475526190

Mersenne Buttons

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Jumping GIFS

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- George Woltman founded GIMPS in January 1996 and wrote the prime testing software.
- Scott Kurowski wrote the PrimeNet server that supports GIMPS. In 1997 he founded Entropia, a distributed computing software company.

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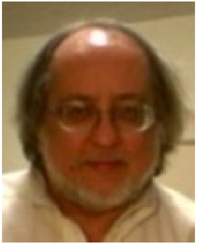
- UCM has over 700 computers performing LL-tests on Mersenne numbers.

GIMPS People

Woltman

Kurowski



Crandall

GIMPS Links

- The GIMPS home page can be found at: https://www.mersenne.org

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- A Mersenne Prime discussion forum can be found at: http://www.mersenneforum.org

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Definition

Let $S_1 = 4$ and

$$S_{n+1} = S_n^2 - 2 \text{ for } n \geq 1.$$

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Definition

Let $S_1 = 4$ and

$$S_{n+1} = S_n^2 - 2 \text{ for } n \geq 1.$$

- The first few terms of the S sequence are:

$$4,\ 14,\ 194,\ 37634,\ 1416317954,\ 2005956546822746114,\ 4023861667741036022825635656102100994, \ldots$$

Lucas-Lehmer Test

Lucas-Lehmer Test

Let p be a prime number. Then

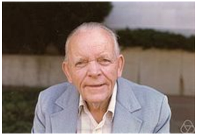
$$M_p = 2^p - 1 \text{ is prime}$$

if and only if

$$S_{p-1} \bmod M_p = 0.$$

Lucas-Lehmer Test

Lucas



Lehmer

Theorem

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Proof

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1	4

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Proof

i	$S_i \bmod 2047$
1	4
2	$(4^2 - 2) = 14 \bmod 2047 = 14$
3	$(14^2 - 2) = 194 \bmod 2047 = 194$

Theorem

$M_{11} = 2^{11} - 1 = 2047$ *is not prime.*

Proof

i	$S_i \bmod 2047$
1	4
2	$(4^2 - 2) = 14 \bmod 2047 = 14$
3	$(14^2 - 2) = 194 \bmod 2047 = 194$
4	$(194^2 - 2) = 37634 \bmod 2047 = 788$

Theorem

$M_{11} = 2^{11} - 1 = 2047$ *is not prime.*

Proof

i	$S_i \bmod 2047$
1	4
2	$(4^2 - 2) = 14 \bmod 2047 = 14$
3	$(14^2 - 2) = 194 \bmod 2047 = 194$
4	$(194^2 - 2) = 37634 \bmod 2047 = 788$
5	$(788^2 - 2) = 620942 \bmod 2047 = 701$

$2^{11} - 1$ is not prime

Proof cont.

i	$S_i \bmod 2047$

$2^{11} - 1$ is not prime

Proof cont.

i	$S_i \bmod 2047$
6	$(701^2 - 2) = 491399 \bmod 2047 = 119$

$2^{11} - 1$ is not prime

Proof cont.

i	$S_i \bmod 2047$
6	$(701^2 - 2) = 491399 \bmod 2047 = 119$
7	$(119^2 - 2) = 14159 \bmod 2047 = 1877$

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Proof cont.

i	$S_i \bmod 2047$
6	$(701^2 - 2) = 491399 \bmod 2047 = 119$
7	$(119^2 - 2) = 14159 \bmod 2047 = 1877$
8	$(1877^2 - 2) = 3523127 \bmod 2047 = 240$

$2^{11} - 1$ is not prime

Proof cont.

i	$S_i \bmod 2047$
6	$(701^2 - 2) = 491399 \bmod 2047 = 119$
7	$(119^2 - 2) = 14159 \bmod 2047 = 1877$
8	$(1877^2 - 2) = 3523127 \bmod 2047 = 240$
9	$(240^2 - 2) = 57598 \bmod 2047 = 282$

$2^{11} - 1$ is not prime

Proof cont.

i	$S_i \bmod 2047$
6	$(701^2 - 2) = 491399 \bmod 2047 = 119$
7	$(119^2 - 2) = 14159 \bmod 2047 = 1877$
8	$(1877^2 - 2) = 3523127 \bmod 2047 = 240$
9	$(240^2 - 2) = 57598 \bmod 2047 = 282$
10	$(282^2 - 2) = 79522 \bmod 2047 = 1736$

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3	194
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1	4
2	14
3	194
4	37634
5	1416317954
6	669670838
7	1937259419
8	425413602

$2^{31} - 1$ is prime

i	$S_i \bmod (2^{31} - 1)$
9	842014276
10	12692426
11	2044502122
12	1119438707
13	1190075270
14	1450757861
15	877666528
16	630853853
17	940321271
18	512995887
19	692931217

$2^{31} - 1$ is prime

i	$S_i \bmod (2^{31} - 1)$
20	1883625615
21	1992425718
22	721929267
23	27220594
24	1570086542
25	1676390412

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26	1159251674
27	211987665

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26	1159251674
27	211987665
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25	1676390412
26	1159251674
27	211987665
28	1181536708
29	65536

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21	1992425718
22	721929267
23	27220594
24	1570086542
25	1676390412
26	1159251674
27	211987665
28	1181536708
29	65536
30	0

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- Based on some theorems Lucas discovered and properties of Fibonacci numbers, his hand calculations boiled down to showing that if $r_1 = 3$, and

$$r_{k+1} = r_k^2 - 2,$$

then if

$$r_{126} \bmod M127 = 0,$$

then M127 is prime.

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- To see how Lucas did this, we will reduce the problem.
- We will show that $M7 = 2^7 - 1 = 127$ is prime.
- For our reduced problem, we will play Lucas' game on a 7×7 chessboard.

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- Therefore, M7 is prime.

- The 7×7 chessboard will store the calculations in base 2 (modulo 127). Columns on the board will represent powers of 2 and the rows will store the product of a single base 2 digit in r_k times the base 2 number r_k. Lucas used a pawn or no pawn to represent a 1 or 0 on the board, respectively.

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- Initially, the top row will contain $r_1 = 3$.

- If the top row contained r_k, Lucas would square r_k with the following moves.

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- He would do standard multiplication to populate the board with pawns. Each row corresponds to putting a shift of the top row in the row or having no pawns in the row, depending on whether there is a pawn in the corresponding column of the top row or not. Because Lucas is doing the calculations modulo 127, the columns wrap around the chessboard.

- He would then subtract 2 (once), usually by taking a pawn away from Column f. In the game, two pawns in the same column would be equivalent to removing those two pawns and replacing them by one pawn in the next column to the left. The column to the left of the left-most column is the right-most column.

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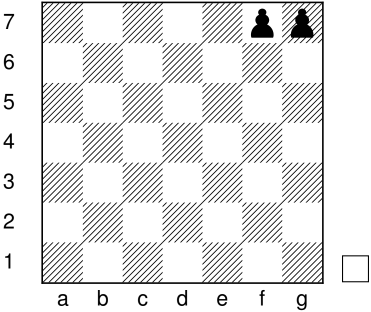
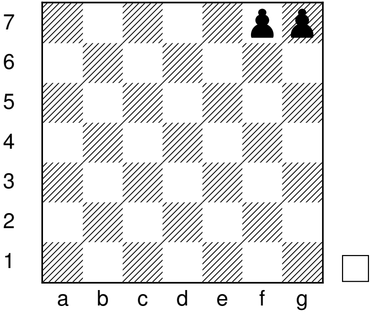
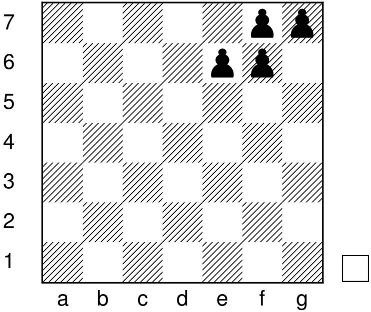
- Lucas kept this game going until he didn't have two pawns in any column. Then he would slide each pawn in a column to the top row. This would be his r_{k+1}.

Lucas started the game with $r_1 = 3$.
On the chessboard, that would be:

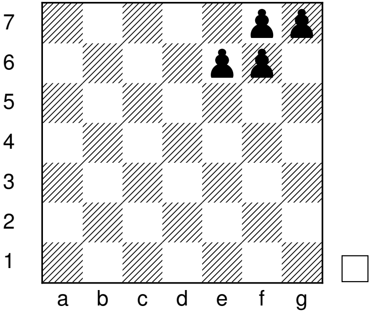


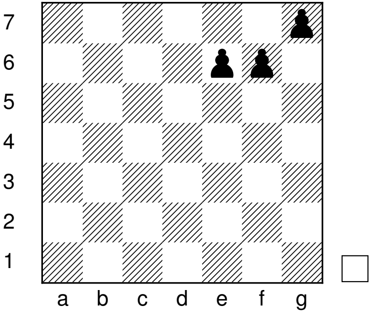
Squaring $r_1 = 3$ would result in the following chessboard.



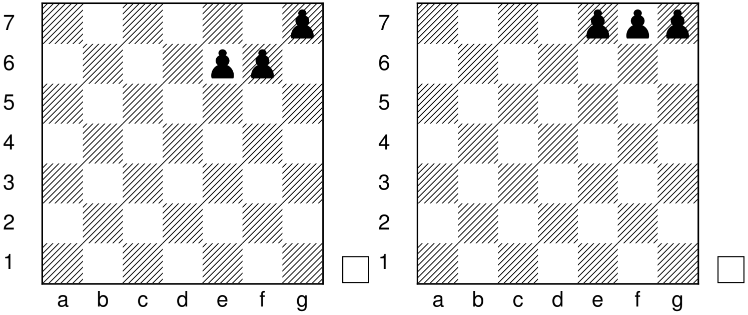


We can subtract 2 by removing a pawn from Column f. That would result in the following chessboard.

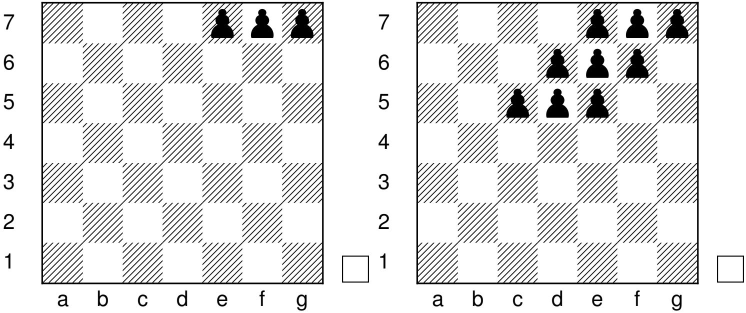




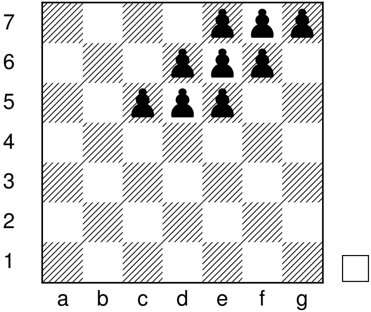
Pushing all the pawns to the top row would result in the following chessboard which is $r_2 = 7$.

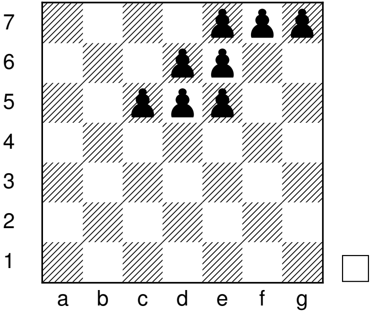


Now we need to square $r_2 = 7$. This would result in the following chessboard.

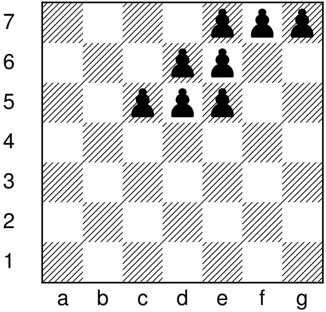


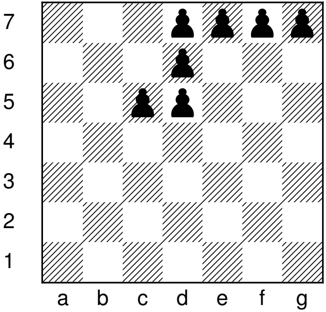
Subtracting 2 would result in the following chessboard.

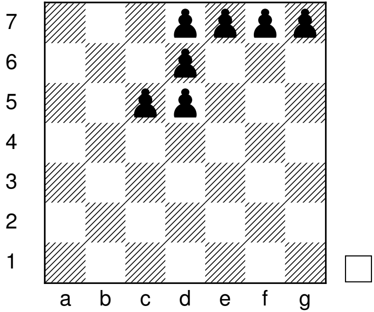


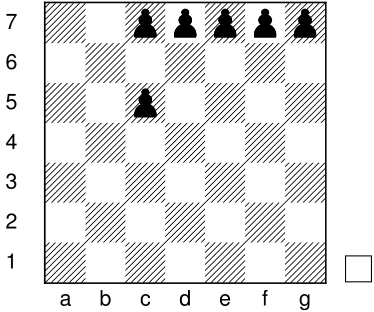


We now do the game moves where we replace two pawns in a column by one pawn in the column to the left. Here are the steps in the game.






7
6
5
4
3
2
1
a b c d e f g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6
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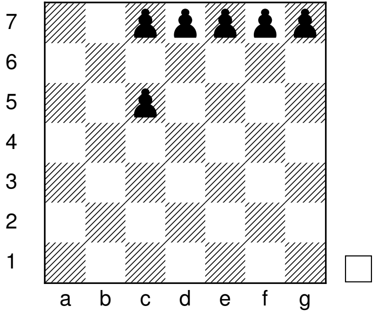
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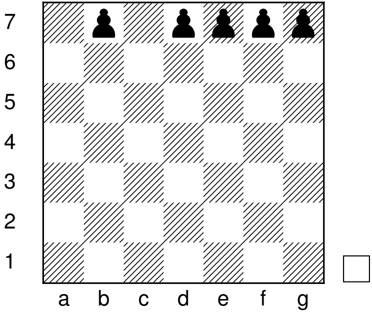
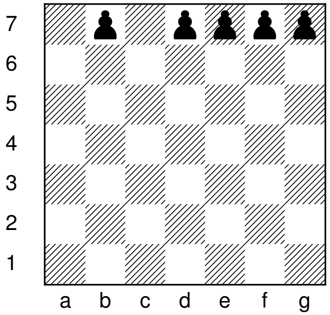
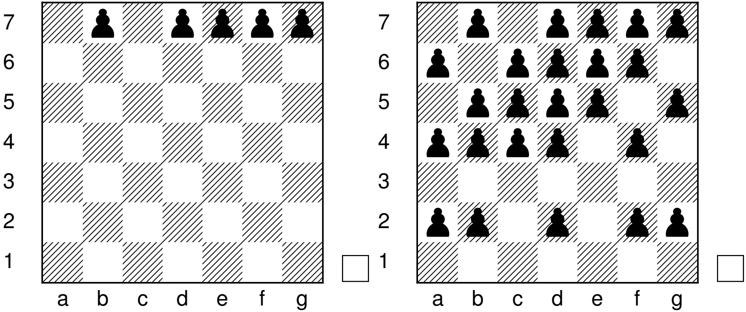

7
6
5
4
3
2
1
a b c d e f g

The final chessboard with $r_3 = 47$ would be the following.



$\square$

Squaring $r_3 = 47$, we obtain the following chessboard.



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Continuing this game, we have $r_4 = 48$, $r_5 = 16$, and $r_6 = 0$. Therefore $M7 = 2^7 - 1 = 127$ is a Mersenne prime.

1. Mersenne Primes
   - Primes
   - Mersenne Primes
2. History of Mersenne Primes
   - Marin Mersenne
   - Edouard Lucas
   - Computer Era
3. 50th Mersenne Prime
   - M77232917
   - News on 50th Mersenne Prime
4. GIMPS
   - GIMPS
   - GIMPS People
   - GIMPS Links
5. Lucas-Lehmer Test and Lucas Game
   - Lucas-Lehmer Test

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6. To help detect hardware problems (fan and CPU/bus problems) on individual computers at UCM.

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4. To learn more about the distribution of Mersenne primes.

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1. To win the $150,000 offered by the Electronic Frontier Foundation (EFF) for the discovery of the first one-hundred million digit prime number. EFF's motivation is to encourage research in computational number theory related to large primes.

Email Address and Talk URL

Curtis Cooper's Email:
cooper@ucmo.edu

Talk:
cs.ucmo.edu/∼cnc8851/talks/gimpsmsa4/mersennemsa4.pdf