

# Hang Chen and Curtis Cooper University of Central Missouri 

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Hang Chen and Curtis Cooper

## Outline

## (1) An Amazing Card Trick

## 2 An Improved "Best Card Trick"

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## Card Tricks

## Amazing Mathematical Card Trick

We have performed the card trick according to Arthur Benjamin's Amazing Mathematical Card Trick.

## Amazing Mathematical Card Trick

We have performed the card trick according to Arthur Benjamin's Amazing Mathematical Card Trick.

We will analyze the card trick by using matrices. To make it easier, when we fold a row or column to the next one, we will flip the cards but leave them in the same positions.

## Face Matrix

For an arrangement of cards, we use $F$ to denote its face matrix.


$$
F=\left(\begin{array}{lllll}
1 & 1 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 & 0 \\
0 & 1 & 1 & 1 & 0 \\
1 & 0 & 1 & 0 & 1
\end{array}\right)
$$


where 0 indicates a face-down card and 1 a face-up card.

## Folding Operators

For an $m \times n$ face matrix, we use two row folding operators to achieve the equivalent row folding as in the magic show.

- Fold row $i$ onto row $i+1$ :

$$
R_{i}^{+}=\left(\begin{array}{cc}
t l_{i} & 0 \\
0 & l
\end{array}\right) m \times m \text { for } i<m
$$

## Folding Operators

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t l_{i} & 0 \\
0 & l
\end{array}\right) m \times m \text { for } i<m .
$$

- Fold row $i$ onto row $i-1$ :

$$
R_{i}^{-}=\left(\begin{array}{cc}
I & 0 \\
0 & t l_{m-i+1}
\end{array}\right) m \times m \text { for } i>1
$$

## Folding Operators

Similarly, we also use two column folding operators.

- Fold column $i$ onto column $i+1$ :

$$
C_{i}^{+}=\left(\begin{array}{cc}
t l_{i} & 0 \\
0 & l
\end{array}\right)_{n \times n} \text { for } i<n .
$$

## Folding Operators

Similarly, we also use two column folding operators.

- Fold column $i$ onto column $i+1$ :

$$
C_{i}^{+}=\left(\begin{array}{cc}
t l_{i} & 0 \\
0 & l
\end{array}\right)_{n \times n} \text { for } i<n .
$$

- Fold column $i$ onto column $i-1$ :

$$
C_{i}^{-}=\left(\begin{array}{cc}
1 & 0 \\
0 & t I_{n-i+1}
\end{array}\right)_{n \times n} \text { for } i>1
$$

## Folding Operators

Here are some special product rules.

- Multiplying an operator and a face matrix:

$$
\begin{aligned}
& t 1=1 t=0 \\
& t 0=0 t=1
\end{aligned}
$$

## Folding Operators

Here are some special product rules.

- Multiplying an operator and a face matrix:

$$
\begin{aligned}
& t 1=1 t=0 \\
& t 0=0 t=1
\end{aligned}
$$

- Multiplying two operators:

$$
t t=1
$$

## (p,q)-Folding

- Lemma: Multiplication of row or column folding operators is commutative.

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Card Tricks

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- A $(p, q)$-folding is that all the cards are folded up and are piled up at the $(p, q)$-position.


## (p,q)-Folding

- Lemma: Multiplication of row or column folding operators is commutative.
- A $(p, q)$-folding is that all the cards are folded up and are piled up at the $(p, q)$-position.
- In fact, a $(p, q)$-folding of $F$ is

$$
R_{1}^{+} \cdots R_{p-1}^{+} R_{p+1}^{-} \cdots R_{m}^{-} F C_{1}^{+} \cdots C_{q-1}^{+} C_{q+1}^{-} \cdots C_{n}^{-}
$$

## A (3, 1)-Folding Example

We show a (3, 1)-folding of $F$. First, fold row one onto row two:

$$
\begin{aligned}
& R_{1}^{+} F= \\
& \quad\left(\begin{array}{llll}
t & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right)\left(\begin{array}{lllll}
1 & 1 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 & 0 \\
0 & 1 & 1 & 1 & 0 \\
1 & 0 & 1 & 0 & 1
\end{array}\right)=\left(\begin{array}{lllll}
0 & 0 & 1 & 0 & 1 \\
0 & 0 & 1 & 1 & 0 \\
0 & 1 & 1 & 1 & 0 \\
1 & 0 & 1 & 0 & 1
\end{array}\right)
\end{aligned}
$$

## A (3, 1)-Folding Example

Next, we fold row four to row three:

$$
\begin{aligned}
& R_{4}^{-} R_{1}^{+} F= \\
& \quad\left(\begin{array}{llll}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & t
\end{array}\right)\left(\begin{array}{lllll}
0 & 0 & 1 & 0 & 1 \\
0 & 0 & 1 & 1 & 0 \\
0 & 1 & 1 & 1 & 0 \\
1 & 0 & 1 & 0 & 1
\end{array}\right)=\left(\begin{array}{lllll}
0 & 0 & 1 & 0 & 1 \\
0 & 0 & 1 & 1 & 0 \\
0 & 1 & 1 & 1 & 0 \\
0 & 1 & 0 & 1 & 0
\end{array}\right)
\end{aligned}
$$

## A (3, 1)-Folding Example

Next, we fold row two to row three:

$$
\begin{aligned}
& R_{2}^{+} R_{4}^{-} R_{1}^{+} F= \\
& \quad\left(\begin{array}{llll}
t & 0 & 0 & 0 \\
0 & t & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right)\left(\begin{array}{lllll}
0 & 0 & 1 & 0 & 1 \\
0 & 0 & 1 & 1 & 0 \\
0 & 1 & 1 & 1 & 0 \\
0 & 1 & 0 & 1 & 0
\end{array}\right)=\left(\begin{array}{lllll}
1 & 1 & 0 & 1 & 0 \\
1 & 1 & 0 & 0 & 1 \\
0 & 1 & 1 & 1 & 0 \\
0 & 1 & 0 & 1 & 0
\end{array}\right)
\end{aligned}
$$

## A (3, 1)-Folding Example

Next, we fold column five to column four:

$$
\begin{aligned}
& R_{2}^{+} R_{4}^{-} R_{1}^{+} F C_{5}^{-}= \\
& \quad\left(\begin{array}{lllll}
1 & 1 & 0 & 1 & 0 \\
1 & 1 & 0 & 0 & 1 \\
0 & 1 & 1 & 1 & 0 \\
0 & 1 & 0 & 1 & 0
\end{array}\right)\left(\begin{array}{lllll}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & t
\end{array}\right)=\left(\begin{array}{lllll}
1 & 1 & 0 & 1 & 1 \\
1 & 1 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 1 \\
0 & 1 & 0 & 1 & 1
\end{array}\right)
\end{aligned}
$$

## A (3, 1)-Folding Example

Next, we fold column four to column three:

$$
\begin{aligned}
& R_{2}^{+} R_{4}^{-} R_{1}^{+} F C_{5}^{-} C_{4}^{-}= \\
& \quad\left(\begin{array}{lllll}
1 & 1 & 0 & 1 & 1 \\
1 & 1 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 1 \\
0 & 1 & 0 & 1 & 1
\end{array}\right)\left(\begin{array}{lllll}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & t & 0 \\
0 & 0 & 0 & 0 & t
\end{array}\right)=\left(\begin{array}{lllll}
1 & 1 & 0 & 0 & 0 \\
1 & 1 & 0 & 1 & 1 \\
0 & 1 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0
\end{array}\right)
\end{aligned}
$$

## A (3, 1)-Folding Example

Next, we fold column three to column two:

$$
\begin{aligned}
& R_{2}^{+} R_{4}^{-} R_{1}^{+} F C_{5}^{-} C_{4}^{-} C_{3}^{-}= \\
& \quad\left(\begin{array}{lllll}
1 & 1 & 0 & 0 & 0 \\
1 & 1 & 0 & 1 & 1 \\
0 & 1 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0
\end{array}\right)\left(\begin{array}{lllll}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & t & 0 & 0 \\
0 & 0 & 0 & t & 0 \\
0 & 0 & 0 & 0 & t
\end{array}\right)=\left(\begin{array}{lllll}
1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 & 1 \\
0 & 1 & 1 & 1 & 1
\end{array}\right)
\end{aligned}
$$

## A (3, 1)-Folding Example

Fold column two to column one to complete $(3,1)$-folding:

$$
R_{2}^{+} R_{4}^{-} R_{1}^{+} F C_{5}^{-} C_{4}^{-} C_{3}^{-} C_{2}^{-}=R_{1}^{+} R_{2}^{+} R_{4}^{-} F C_{2}^{-} C_{3}^{-} C_{4}^{-} C_{5}^{-}=
$$

$$
\left(\begin{array}{lllll}
1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 & 1 \\
0 & 1 & 1 & 1 & 1
\end{array}\right)\left(\begin{array}{lllll}
1 & 0 & 0 & 0 & 0 \\
0 & t & 0 & 0 & 0 \\
0 & 0 & t & 0 & 0 \\
0 & 0 & 0 & t & 0 \\
0 & 0 & 0 & 0 & t
\end{array}\right)=\left(\begin{array}{lllll}
1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 1 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{array}\right)
$$

## A (3,1)-Folding Example

$$
\left(\begin{array}{lllll}
1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 1 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{array}\right)
$$



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## Card Tricks

## Observations on (3, 1)-Folding

Let $R=R_{1}^{+} R_{2}^{+} R_{4}^{-}$and $C=C_{2}^{-} C_{3}^{-} C_{4}^{-} C_{5}^{-}$. Then,

$$
R F C=\left(\begin{array}{lllll}
1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 1 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{array}\right) .
$$

## Observations on (3, 1)-Folding

In fact,

$$
R=\left(\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & t & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & t
\end{array}\right) \text { and } C=\left(\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & t & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & t
\end{array}\right)
$$

## Observations on (3, 1)-Folding

In fact,

$$
R=\left(\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & t & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & t
\end{array}\right) \text { and } C=\left(\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & t & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & t
\end{array}\right)
$$

- As $R$ acts, a card is flipped if and only if it is in an even row.


## Observations on (3, 1)-Folding

In fact,

$$
R=\left(\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & t & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & t
\end{array}\right) \text { and } C=\left(\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & t & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & t
\end{array}\right)
$$

- As $R$ acts, a card is flipped if and only if it is in an even row.
- As $C$ acts, a card is flipped if and only if it is in an even column.


## Observations on (3, 1)-Folding

- Therefore, as both $R$ and $C$ act on F , that is in $R F C$, a card at position $(i, j)$ is flipped if and only if $i+j$ is odd.


## Observations on (3, 1)-Folding

- Therefore, as both $R$ and $C$ act on $F$, that is in RFC, a card at position $(i, j)$ is flipped if and only if $i+j$ is odd.
- Namely, a card is flipped if and only if it is in an even row and odd column or an odd row and even column.


## Observations on (3, 1)-Folding



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## Card Tricks

## Results

## Lemma: For a $(p, q)$-folding, $R=\operatorname{diag}[t, 1, t, 1, \ldots]$ for even $p$ and $R=\operatorname{diag}[1, t, 1, t, \ldots]$ for odd $p$.

## Results

Lemma: For a $(p, q)$-folding, $R=\operatorname{diag}[t, 1, t, 1, \ldots]$ for even $p$ and $R=\operatorname{diag}[1, t, 1, t, \ldots]$ for odd $p$.

Lemma: For a $(p, q)$-folding, $C=\operatorname{diag}[t, 1, t, 1, \ldots]$ for even $q$ and $C=\operatorname{diag}[1, t, 1, t, \ldots]$ for odd $q$.

## Results

- The position parity of a card at $(i, j)$ is defined as the parity of $i+j$.


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- The parity of $(p, q)$-folding is defined as the parity of $p+q$.


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- The position parity of a card at $(i, j)$ is defined as the parity of $i+j$.
- The parity of $(p, q)$-folding is defined as the parity of $p+q$.
- Theorem: $\mathrm{A}(p, q)$-folding flips a card if and only if the card's position parity is opposite to the folding parity.


## Results

- The position parity of a card at $(i, j)$ is defined as the parity of $i+j$.
- The parity of $(p, q)$-folding is defined as the parity of $p+q$.
- Theorem: $\mathrm{A}(p, q)$-folding flips a card if and only if the card's position parity is opposite to the folding parity.
- In the (3, 1)-folding example, a card is not flipped if and only if it is at an even position.


## Results

An even arrangement:

- Face-up hearts and face-down non-hearts at even positions

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## Results

An even arrangement:

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- Face-down hearts and face-up non-hearts at odd positions


## Results

An even arrangement:

- Face-up hearts and face-down non-hearts at even positions
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An odd arrangement:

- Face-up hearts and face-down non-hearts at odd positions


## Results

An even arrangement:

- Face-up hearts and face-down non-hearts at even positions
- Face-down hearts and face-up non-hearts at odd positions

An odd arrangement:

- Face-up hearts and face-down non-hearts at odd positions
- Face-down hearts and face-up non-hearts at even positions


## Results

- Corollary: If the parities of the arrangement and the folding are the same, then the resulting array of cards has all hearts face up and all non-hearts face down.


## Results

- Corollary: If the parities of the arrangement and the folding are the same, then the resulting array of cards has all hearts face up and all non-hearts face down.
- Corollary: If the parities of the arrangement and the folding are the opposite, then the resulting array of cards has all hearts face down and all non-hearts face up.


## Making an Even Arrangement of Cards

In the earlier show, we made an even arrangement of cards.

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With odd number of columns, out of a pair of cards, the top card will always be at an even position and the bottom card at an odd position.

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With odd number of columns, out of a pair of cards, the top card will always be at an even position and the bottom card at an odd position.

- Two hearts: back-to-back


## Making an Even Arrangement of Cards

In the earlier show, we made an even arrangement of cards.
With odd number of columns, out of a pair of cards, the top card will always be at an even position and the bottom card at an odd position.

- Two hearts: back-to-back
- Two non-hearts: face-to-face


## Making an Even Arrangement of Cards

In the earlier show, we made an even arrangement of cards.
With odd number of columns, out of a pair of cards, the top card will always be at an even position and the bottom card at an odd position.

- Two hearts: back-to-back
- Two non-hearts: face-to-face
- One heart and one non-heart: either both face-up with heart on top


## Making an Even Arrangement of Cards

In the earlier show, we made an even arrangement of cards.
With odd number of columns, out of a pair of cards, the top card will always be at an even position and the bottom card at an odd position.

- Two hearts: back-to-back
- Two non-hearts: face-to-face
- One heart and one non-heart: either both face-up with heart on top or both face-down with heart on the bottom.


## More Examples

Example: According to even arrangement, we put 12 hearts in a $4 \times 3$ array. A (2, 2)-folding results in all cards face up.

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$$
\begin{aligned}
& \left(\begin{array}{llll}
t & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & t & 0 \\
0 & 0 & 0 & 1
\end{array}\right)\left(\begin{array}{lll}
1 & 0 & 1 \\
0 & 1 & 0 \\
1 & 0 & 1 \\
0 & 1 & 0
\end{array}\right)\left(\begin{array}{lll}
t & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & t
\end{array}\right) \\
& =\left(\begin{array}{lll}
0 & 1 & 0 \\
0 & 1 & 0 \\
0 & 1 & 0 \\
0 & 1 & 0
\end{array}\right)\left(\begin{array}{lll}
t & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & t
\end{array}\right)=\left(\begin{array}{lll}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1
\end{array}\right)
\end{aligned}
$$

## More Examples

Example: A few cards in a non-complete array and its face matrix.


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## Card Tricks

## More Examples

A (3,4)-folding will result in all hearts face down.

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A (3,4)-folding will result in all hearts face down.

$$
\left(\begin{array}{llll}
1 & 0 & 0 & 0 \\
0 & t & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & t
\end{array}\right)\left(\begin{array}{lllll}
1 & 1 & 0 & & \\
0 & & 1 & 1 & 0 \\
0 & 1 & 1 & & \\
& 0 & & &
\end{array}\right)\left(\begin{array}{lllll}
t & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & t & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & t
\end{array}\right)=
$$

## More Examples



All face-down cards are hearts.

## Card Tricks

## Outline

## (1) An Amazing Card Trick

## 2 An Improved "Best Card Trick"

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## Card Tricks

## The Best Card Trick - Fitch Cheney

Fitch Cheney asks you to pick five cards. Two cards will be of the same suit. Here is an example:


## The Best Card Trick - Fitch Cheney

Fitch Cheney asks you to pick five cards. Two cards will be of the same suit. Here is an example:


Then he has to give you the nine of hearts to hide from his assistant (audience might not like this) and places the rest of four cards in this way:


## The Best Card Trick - Fitch Cheney



- The first card, three of hearts (the hidden card's suit), indicates the hidden card is a heart.


## The Best Card Trick - Fitch Cheney



- The first card, three of hearts (the hidden card's suit), indicates the hidden card is a heart.
- The next three cards form a permutation (six) to indicate the hidden card's rank is six higher than three (the first card's rank).


## The Best Card Trick - Fitch Cheney

- There are six permutations of $\{L, M, H\}$ :

$$
\begin{aligned}
& L M H=1, L H M=2, M L H=3 \\
& M H L=4, H L M=5, H M L=6
\end{aligned}
$$

## The Best Card Trick - Fitch Cheney

- There are six permutations of $\{L, M, H\}$ :

$$
\begin{aligned}
& L M H=1, L H M=2, M L H=3 \\
& M H L=4, H L M=5, H M L=6
\end{aligned}
$$

- Cheney orders the whole deck of cards in this way:
 2円, ..., Kゅ.


## The Best Card Trick - Fitch Cheney

Therefore,

indicates $6=\mathrm{HML}$.

He had to hide the nine of hearts (instead of the three of hearts) since only six permutations are available.

## Improvements

We don't like the fact that Cheney dictates which card to hide instead of letting the audience choose.

## Improvements

We don't like the fact that Cheney dictates which card to hide instead of letting the audience choose.

We don't like the way Cheney orders the cards.

## Improvements

We let audience pick six cards and decide which card to hide.


## Improvements

We let audience pick six cards and decide which card to hide.


We arrange the remaining five cards:


## Improvements

We define the increasing order of the cards as follows:

- The increasing order of cards is given by the increasing ranks first: $2<3<\cdots<K<A$.


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We define the increasing order of the cards as follows:

- The increasing order of cards is given by the increasing ranks first: $2<3<\cdots<K<A$.
- Then within a rank the order of suits is $\boldsymbol{\AA}<\diamond<\ominus<\boldsymbol{\phi}$ (Natural to bridge players).


## Improvements

We define the increasing order of the cards as follows:

- The increasing order of cards is given by the increasing ranks first: $2<3<\cdots<K<A$.
- Then within a rank the order of suits is $\boldsymbol{\omega}<\diamond<\diamond<\boldsymbol{\omega}$ (Natural to bridge players).
- The whole deck of cards in this order:

$$
\begin{aligned}
& 2 \&<2 \diamond<2 ๑<2 \uparrow<3 \%<3 \diamond<3 \bigcirc<3 \uparrow<\cdots< \\
& \mathrm{K} \boldsymbol{\&}<\mathrm{K} \diamond<\mathrm{K} \oslash<\mathrm{K} \boldsymbol{\phi}<\mathrm{A} \boldsymbol{\phi}<\mathrm{A} \diamond<\mathrm{A} \bigcirc<\mathrm{A} \boldsymbol{\phi}
\end{aligned}
$$

## Improvements



- The first card, queen of diamonds - the fourth smallest card of the five, indicates the hidden card's suit is the fourth suit - $\boldsymbol{\omega}$, due to $\boldsymbol{\alpha}<\diamond<\diamond<\boldsymbol{\omega}$.


## Improvements



- The first card, queen of diamonds - the fourth smallest card of the five, indicates the hidden card's suit is the fourth suit - $\boldsymbol{\oplus}$, due to $\boldsymbol{\alpha}<\diamond<\ominus<\boldsymbol{\omega}$.
- The second card, three of diamonds - the smallest card of the remaining four, indicates the hidden card's rank is lower than the first card's rank.


## Improvements



- The remaining three cards (HML) indicates six, that is, the hidden card's rank is 6 less than the first card's.


## Improvements



- The remaining three cards (HML) indicates six, that is, the hidden card's rank is 6 less than the first card's.
- Therefore the hidden card is six of spades.


## Improvements

Here is another example:


- The audience hides the jack of spades. We arrange the five cards in this way:



## Improvements



- In this case, the first card (indicating the hidden card is a spade) is of the same rank of the hidden card. It seems that we have a harder time. In fact, we made it easier.


## Improvements



- In this case, the first card (indicating the hidden card is a spade) is of the same rank of the hidden card. It seems that we have a harder time. In fact, we made it easier.
- We only need to put either jack of clubs or six of diamonds as the second card to indicate the rank of the hidden card is NOT larger than OR smaller than the first card's.


## Improvements



- By now, we already know the hidden card is jack of spades.


## Improvements



- By now, we already know the hidden card is jack of spades.
- Therefore, the remaining cards don't matter - we put them in casually.


## Problem 1



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## Card Tricks

## Solution 1



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## Card Tricks

## Problem 2



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## Solution 2

## $\stackrel{A}{\square}$



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## Card Tricks

## Problem 3



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## Solution 3



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## Problem 4



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## Solution 4



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