Algebraic Statements Similar to Those in Ramanujan's "Lost Notebook"

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Outline

In his “lost notebook”, Ramanujan gave twelve algebraic statements. Here are some of these statements.

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If $g^5 = 3$, then

$$\frac{\sqrt{g^2+1}+\sqrt{5g-5}}{\sqrt{g^2+1}-\sqrt{5g-5}} = \frac{1}{g} + g + g^2 + g^3. \tag{1}$$

In his “lost notebook”, Ramanujan gave twelve algebraic statements. Here are some of these statements.

If $g^5 = 3$, then

$$\frac{\sqrt{g^2+1}+\sqrt{5g-5}}{\sqrt{g^2+1}-\sqrt{5g-5}} = \frac{1}{g} + g + g^2 + g^3. \tag{1}$$

If $g^5 = 2$, then

$$\sqrt{1+g^2} = \frac{g^4+g^3+g-1}{\sqrt{5}}. \tag{2}$$

If $g^5 = 2$, then

$$\sqrt{4g - 3} = \frac{g^9 + g^7 - g^6 - 1}{\sqrt{5}}. \tag{3}$$

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If $g^5 = 3$, then

$$\sqrt[3]{2-g^3} = \frac{1 + g - g^2}{\sqrt[3]{5}}. \tag{4}$$

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If $g^4 = 5$, then

$$\frac{\sqrt[5]{3+2g} - \sqrt[5]{4-4g}}{\sqrt[5]{3+2g} + \sqrt[5]{4-4g}} = 2 + g + g^2 + g^3. \tag{6}$$

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One technique he used and we will use throughout the paper is *componendo et dividendo*, which states that

$$\frac{a+b}{a-b} = \frac{c+d}{c-d} \text{ if and only if } \frac{a}{b} = \frac{c}{d}.$$

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One technique he used and we will use throughout the paper is *componendo et dividendo*, which states that

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We will give some similar algebraic statements to Ramanujan's statements and provide some proofs.

Outline

Here is Ramanujan's algebraic statement (1).
If $g^5 = 3$, then

$$\frac{\sqrt{g^2+1}+\sqrt{5g-5}}{\sqrt{g^2+1}-\sqrt{5g-5}} = \frac{1}{g} + g + g^2 + g^3.$$

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Here is a similar statement.

Theorem

If $g^5 = 2$, then

$$\frac{\sqrt{4g^2+g+2}+\sqrt{8g^2+41g-54}}{\sqrt{4g^2+g+2}-\sqrt{8g^2+41g-54}} = \frac{1}{g} + g + g^2 + g^3.$$

Theorem

If $g^5 = 2$*, then*

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Proof.

The algebraic statement is equivalent to

$$\frac{\sqrt{4g^2+g+2}+\sqrt{8g^2+41g-54}}{\sqrt{4g^2+g+2}-\sqrt{8g^2+41g-54}} = \frac{1+g^2+g^3+g^4}{g}.$$

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$$\frac{\sqrt{4g^2+g+2}+\sqrt{8g^2+41g-54}}{\sqrt{4g^2+g+2}-\sqrt{8g^2+41g-54}} = \frac{1+g^2+g^3+g^4}{g}.$$

Thus, by *componendo et dividendo*, we need to show that

$$\sqrt{\frac{4g^2+g+2}{8g^2+41g-54}} = \frac{1+g+g^2+g^3+g^4}{1-g+g^2+g^3+g^4}.$$

This is equivalent to showing that

$$(1+g+g^2+g^3+g^4)^2(8g^2+41g-54)$$
$$=(1-g+g^2+g^3+g^4)^2(4g^2+g+2).$$

This is equivalent to showing that

$$(1+g+g^2+g^3+g^4)^2(8g^2+41g-54)$$
$$=(1-g+g^2+g^3+g^4)^2(4g^2+g+2).$$

Expanding both sides of the above equation and using the fact that $g^5=2$, the left- and right-hand sides of the equation are equal and the theorem is proved.

Outline

Here is Ramanujan's algebraic statement (2).
If $g^5 = 2$, then

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Theorem

If $g^5 = 8$, then

$$\sqrt{2g^2-3} = \frac{g^4+2g^3-2g^2-2}{2\sqrt{5}}.$$

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$$\sqrt{2g^2 - 3} = \frac{g^4 + 2g^3 - 2g^2 - 2}{2\sqrt{5}}.$$

Proof.

Using the fact that $g^5 = 8$, we have the following equalities.

$$\begin{aligned}(g^4 + 2g^3 - 2g^2 - 2)^2 &= g^8 + 4g^7 - 8g^5 - 8g^3 + 8g^2 + 4 \\ &= 8g^3 + 32g^2 - 64 - 8g^3 + 8g^2 + 4 \\ &= 40g^2 - 60 = 20(2g^2 - 3).\end{aligned}$$

Theorem

If $g^5 = 8$*, then*

$$\sqrt{2g^2 - 3} = \frac{g^4 + 2g^3 - 2g^2 - 2}{2\sqrt{5}}.$$

Proof.

Using the fact that $g^5 = 8$, we have the following equalities.

$$\begin{aligned}(g^4 + 2g^3 - 2g^2 - 2)^2 &= g^8 + 4g^7 - 8g^5 - 8g^3 + 8g^2 + 4 \\ &= 8g^3 + 32g^2 - 64 - 8g^3 + 8g^2 + 4 \\ &= 40g^2 - 60 = 20(2g^2 - 3).\end{aligned}$$

This proves the theorem.

To find some more algebraic identities similar to (2), we used a C++ program to search for solutions to

$$(R + Sg + Tg^2 + Ug^3 + Vg^4)^2 = Cg^2 + E,$$

where $g^5 = h + 1$.

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$$(R + Sg + Tg^2 + Ug^3 + Vg^4)^2 = Cg^2 + E,$$

where $g^5 = h + 1$.
Here is the C++ program.

```
#include <iostream>
#include <fstream>

using namespace std;

// This program searches for integers R, S, T,
// U, V, and h
// such that if g^5 = h+1, then
// (R+Sg+Tg^2+Ug^3+Vg^4)^2 = Cg^2 + E
// Bounds for R, S, T, U, V, and h
// will be input from the keyboard.
```

```
long long gcd(long long a, long long b) {
  long long r = a%b;
  while (r>0) {
    a = b;
    b = r;
    r = a%b;
  }
  return b;
}
```

```
int main()
{
  long long upperR, upperS, upperT, upperU;
  long long upperV, upperh;
  long long R, S, T, U, V, A, B, C, D, E, G, h;
  ofstream outfile("outputproggen1_5");
  cout << "Input␣upper␣limit␣for␣R:␣";
  cin >> upperR;
  cout << "Input␣upper␣limit␣for␣S:␣";
  cin >> upperS;
  cout << "Input␣upper␣limit␣for␣T:␣";
  cin >> upperT;
```

```
cout << "Input␣upper␣limit␣for␣U:␣";
cin >> upperU;
cout << "Input␣upper␣limit␣for␣V:␣";
cin >> upperV;
cout << "Input␣upper␣limit␣for␣h:␣";
cin >> upperh;
```

```
cout << "R␣=␣" << upperR << endl;
cout << "S␣=␣" << upperS << endl;
cout << "T␣=␣" << upperT << endl;
cout << "U␣=␣" << upperU << endl;
cout << "V␣=␣" << upperV << endl;
cout << "h␣=␣" << upperh << endl;
outfile << "R␣=␣" << upperR << endl;
outfile << "S␣=␣" << upperS << endl;
outfile << "T␣=␣" << upperT << endl;
outfile << "U␣=␣" << upperU << endl;
outfile << "V␣=␣" << upperV << endl;
outfile << "h␣=␣" << upperh << endl;
```

```
for (R=-upperR; R<=upperR; R++) {
  for (S=-upperS; S<=upperS; S++) {
  for (T=-upperT; T<=upperT; T++) {
    for (U=-upperU; U<=upperU; U++) {
      for (V=1; V<=upperV; V++) {
        for (h=-upperh; h<=upperh; h++) {
```

```
A = 2*R*V+2*S*U+T*T;
B = (h+1)*V*V+2*S*T+2*R*U;
C = 2*(h+1)*U*V+2*R*T+S*S;
D = (h+1)*(U*U+2*T*V)+2*R*S;
E = (h+1)*(2*S*V+2*T*U)+R*R;
if (R!=0 && A==0 && B==0 && h!=-1 && h!=0 && D==0)
```

```
if (R>0) G = R;
else G = -R;
if (S>0) G = gcd(G,S);
else if (S<0) G = gcd(G,-S);
if (T>0) G = gcd(G,T);
else if (T<0) G = gcd(G,-T);
if (U>0) G = gcd(G,U);
else if (U<0) G = gcd(G,-U);
G = gcd(G,V);
if (G==1) {
```

```
cout << "************************" << endl;
cout << "R␣=␣" << R << "␣S␣=␣" << S << endl;
cout << "T␣=␣" << T << "␣U␣=␣" << U << endl;
cout << "V␣=␣" << V << "␣h␣=␣" << h << endl;
cout << "If␣g^5␣=␣" << (h+1) << ",␣then" << endl;
cout << "The␣square␣root␣of␣(" << C
   << "g^2␣+␣" << E << ")" << endl;
cout << "=␣(" << R << "+␣" << S << "g␣+␣"
   << T << "g^2␣+␣" << U << "g^3␣+␣"
   << V << "g^4)␣" << endl;
```

```
outfile << "************************" << endl;
outfile << "R␣=␣" << R << "␣S␣=␣" << S << endl;
outfile << "T␣=␣" << T << "␣U␣=␣" << U << endl;
outfile << "V␣=␣" << V << "␣h␣=␣" << h << endl;
outfile << "If␣g^5␣=␣" << (h+1)
   << ",␣then" << endl;
outfile << "The␣square␣root␣of␣(" << C
   << "g^2␣+␣" << E << ")" << endl;
outfile << "=␣(" << R << "+␣" << S << "g␣+␣"
   << T << "g^2␣+␣" << U << "g^3␣+␣"
   << V << "g^4)␣" << endl;
```

```
                }
                 }
                 }
              }
           }
        }
        }
        }

        return 0;
     }
```

```
R = 20
S = 20
T = 20
U = 20
V = 20
h = 20
************************
R = -9 S = -3
T = -6 U = 3
V = 1 h = 17
If g^5 = 18, then
The square root of (225g^2 + -675)
= (-9+ -3g + -6g^2 + 3g^3 + 1g^4)
```

```
***********************
R = -2 S = 0
T = -2 U = 2
V = 1 h = 7
If g^5 = 8, then
The square root of (40g^2 + -60)
= (-2+ 0g + -2g^2 + 2g^3 + 1g^4)
***********************
R = -1 S = 1
T = 0 U = 1
V = 1 h = 1
If g^5 = 2, then
The square root of (5g^2 + 5)
= (-1+ 1g + 0g^2 + 1g^3 + 1g^4)
```

Using this program, we found the following theorems.

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Theorem

If $g^5 = 18$, *then*

$$\sqrt{g^2 - 3} = \frac{g^4 + 3g^3 - 6g^2 - 3g - 9}{15}.$$

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Theorem

If $g^5 = 18$*, then*

$$\sqrt{g^2 - 3} = \frac{g^4 + 3g^3 - 6g^2 - 3g - 9}{15}.$$

Theorem

If $g^5 = 49$*, then*

$$\sqrt{392g^2 - 343} = \frac{6g^4 + 14g^3 - 14g^2 + 14g - 49}{5}.$$

Outline

Here is Ramanujan's algebraic statement (3).
If $g^5 = 2$, then

$$\sqrt{4g-3} = \frac{g^9 + g^7 - g^6 - 1}{\sqrt{5}}.$$

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If $g^5 = 2$, then

$$\sqrt{4g-3} = \frac{g^9 + g^7 - g^6 - 1}{\sqrt{5}}.$$

Here is a similar statement.

Theorem

If $g^5 = 8$, then

$$\sqrt{g+2} = \frac{g^4 - g^3 + 4g + 4}{2\sqrt{10}}.$$

To find some more algebraic identities similar to (3), we used a C++ program to search for solutions to

$$(R + Sg + Tg^2 + Ug^3 + Vg^4)^2 = Dg + E,$$

where $g^5 = h + 1$. We found the following theorems.

To find some more algebraic identities similar to (3), we used a C++ program to search for solutions to

$$(R + Sg + Tg^2 + Ug^3 + Vg^4)^2 = Dg + E,$$

where $g^5 = h + 1$. We found the following theorems.

Theorem

If $g^5 = 12$, then

$$\sqrt{11g - 7} = \frac{g^4 - g^3 + 2g^2 - 8g - 10}{2\sqrt{5}}.$$

Theorem

If $g^5 = 4$*, then*

$$\sqrt{g+1} = \frac{g^4 + g^3 + 2g^2 - 2}{2\sqrt{5}}.$$

Theorem

If $g^5 = 4$, *then*

$$\sqrt{g+1} = \frac{g^4 + g^3 + 2g^2 - 2}{2\sqrt{5}}.$$

Theorem

If $g^5 = 7$, *then*

$$\sqrt{-g+8} = \frac{2g^4 - g^3 - 2g^2 + 6g + 2}{5}.$$

Outline

Here is Ramanujan's algebraic statement (2).
If $g^5 = 2$, then

$$\sqrt{1+g^2} = \frac{g^4 + g^3 + g - 1}{\sqrt{5}}.$$

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If $g^5 = 2$, then

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If $g^5 = 2$, then

$$\sqrt{4g-3} = \frac{g^9+g^7-g^6-1}{\sqrt{5}}.$$

Here are two similar statements.

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Theorem

If $g^5 = 2$, *then*

$$\sqrt{8g^2 - 20g + 17} = g^9 - g^7 + g^6 - 1.$$

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Theorem

If $g^5 = 2$, *then*

$$\sqrt{8g^2 - 20g + 17} = g^9 - g^7 + g^6 - 1.$$

Theorem

If $g^5 = 2$, *then*

$$\sqrt{-3g^2 + 4g + 5} = g^4 - g^3 + g + 1.$$

Outline

Here is Ramanujan's algebraic statement (4).
If $g^5 = 3$, then

$$\sqrt[3]{2 - g^3} = \frac{1 + g - g^2}{\sqrt[3]{5}}.$$

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If $g^5 = 3$, then

$$\sqrt[3]{2 - g^3} = \frac{1 + g - g^2}{\sqrt[3]{5}}.$$

Here is a similar statement.

Theorem

If $g^5 = 96$, then

$$\sqrt[3]{16 - g^3} = \frac{4 + 2g - g^2}{2\sqrt[3]{5}}.$$

Here is another similar statement.

Theorem

If $g^5 = 3072$, then

$$\sqrt[3]{128 - g^3} = \frac{16 + 4g - g^2}{4\sqrt[3]{5}}.$$

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If $g^5 = 3072$, then

$$\sqrt[3]{128 - g^3} = \frac{16 + 4g - g^2}{4\sqrt[3]{5}}.$$

We were then able to generalize the above results.

Theorem

Let k be a nonnegative integer. If $g^5 = 3 \cdot 32^k$, then

$$\sqrt[3]{2 \cdot 8^k - g^3} = \frac{4^k + 2^k g - g^2}{2^k \sqrt[3]{5}}.$$

Outline

Here is Ramanujan's algebraic statement (5).
If $g^5 = 2$, then

$$\sqrt[5]{1+g+g^3} = \frac{\sqrt{1+g^2}}{\sqrt[10]{5}},$$

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If $g^5 = 2$, then

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Here is a similar statement.

Theorem

If $g^5 = 8$, then

$$\sqrt[5]{2+2g+g^2} = \frac{\sqrt{2+g}}{\sqrt[10]{10}}.$$

To find more algebraic identities similar to (5), we used a C++ program to search for solutions to

$$(R + Sg + Tg^2)^5 = (A + Bg + Cg^2 + Dg^3)^2,$$

where $g^5 = h + 1$. If we have this result, we have more theorems. The next theorem is one that we discovered. The proof is similar to the proof above.

To find more algebraic identities similar to (5), we used a C++ program to search for solutions to

$$(R + Sg + Tg^2)^5 = (A + Bg + Cg^2 + Dg^3)^2,$$

where $g^5 = h + 1$. If we have this result, we have more theorems. The next theorem is one that we discovered. The proof is similar to the proof above.

Theorem

If $g^5 = 8$, then

$$\sqrt[5]{1+g} = \frac{\sqrt{2+g}}{\sqrt[10]{40}}.$$

Outline

We next set out to find algebraic statements similar to (6).
If $g^4 = 5$, then

$$\frac{\sqrt[5]{3+2g} - \sqrt[5]{4-4g}}{\sqrt[5]{3+2g} + \sqrt[5]{4-4g}} = 2 + g + g^2 + g^3.$$

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If $g^4 = 5$, then

$$\frac{\sqrt[5]{3+2g} - \sqrt[5]{4-4g}}{\sqrt[5]{3+2g} + \sqrt[5]{4-4g}} = 2 + g + g^2 + g^3.$$

To construct such a result, we want to find integers h, A, B, C, D, and E so that if $g^4 = h + 1$, then

$$\sqrt[5]{\frac{Ag+B}{Cg+D}} = \frac{E+1+g+g^2+g^3}{1+g+g^2+g^3}.$$

We next set out to find algebraic statements similar to (6).
If $g^4 = 5$, then

$$\frac{\sqrt[5]{3+2g} - \sqrt[5]{4-4g}}{\sqrt[5]{3+2g} + \sqrt[5]{4-4g}} = 2 + g + g^2 + g^3.$$

To construct such a result, we want to find integers h, A, B, C, D, and E so that if $g^4 = h + 1$, then

$$\sqrt[5]{\frac{Ag+B}{Cg+D}} = \frac{E+1+g+g^2+g^3}{1+g+g^2+g^3}.$$

We found the following solutions.

C	D	E	h	A	B
4	-4	2	4	2	3
1	2	-79	79	512	-1536
4	12	-202	404	486	-2187

The first line is Ramanujan's (6). Here are the other theorems.

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Theorem

If $g^4 = 80$, then

$$\frac{\sqrt[5]{512g - 1536} + \sqrt[5]{g+2}}{\sqrt[5]{512g - 1536} - \sqrt[5]{g+2}} = \frac{-77 + 2g + 2g^2 + 2g^3}{-79}.$$

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If $g^4 = 80$, *then*

$$\frac{\sqrt[5]{512g - 1536} + \sqrt[5]{g+2}}{\sqrt[5]{512g - 1536} - \sqrt[5]{g+2}} = \frac{-77 + 2g + 2g^2 + 2g^3}{-79}.$$

Theorem

If $g^4 = 405$, *then*

$$\frac{\sqrt[5]{486g - 2187} + \sqrt[5]{4g+12}}{\sqrt[5]{486g - 2187} - \sqrt[5]{4g+12}} = \frac{-200 + 2g + 2g^2 + 2g^3}{-202}.$$

Outline

We next state some theorems similar to some of Ramanujan's algebraic statements.

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Theorem

If $g^3 = 2$, *then*

$$\frac{\sqrt[4]{111-87g}+\sqrt[4]{g-1}}{\sqrt[4]{111-87g}-\sqrt[4]{g-1}} = 2+g+g^2.$$

We next give another algebraic statement theorem and proof.

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Theorem

If $g^5 = 4$, *then*

$$\frac{3\sqrt{3g^2+4g+6}+\sqrt{55g^2+40g-50}}{3\sqrt{3g^2+4g+6}-\sqrt{55g^2+40g-50}} = \frac{2+2g+g^2+2g^3+2g^4}{g^2}.$$

Here is another algebraic statement.

Here is another algebraic statement.

Theorem

If $g^5 = 2$, *then*

$$\frac{\sqrt[3]{5g^2+1}+\sqrt[3]{35g^2+g-43}}{\sqrt[3]{5g^2+1}-\sqrt[3]{35g^2+g-43}} = \frac{2+g+2g^2+2g^3+2g^4}{g}.$$

Here is another algebraic statement.

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Theorem

If $g^7 = 2$*, then*

$$\frac{\sqrt[5]{12+15g+11g^2+15g^3} + \sqrt[5]{14+315g+346g^2-259g^3-270g^4}}{\sqrt[5]{12+15g+11g^2+15g^3} - \sqrt[5]{14+315g+346g^2-259g^3-270g^4}}$$
$$= \frac{2+g+2g^2+2g^3+2g^4+2g^5+2g^6}{g}.$$

Here is another algebraic statement.

Theorem

If $g^7 = 2$, then

$$\frac{\sqrt[5]{12+15g+11g^2+15g^3} + \sqrt[5]{14+315g+346g^2-259g^3-270g^4}}{\sqrt[5]{12+15g+11g^2+15g^3} - \sqrt[5]{14+315g+346g^2-259g^3-270g^4}} = \frac{2+g+2g^2+2g^3+2g^4+2g^5+2g^6}{g}.$$

One open question is the following.
Can we generate theorems with the hypothesis that $g^k = 2$, where $k \geq 9$ is an odd integer?

Here are some other theorems.

Theorem

If $2g^3 - 3g^2 - 4 = 0$, *then*

$$\frac{\sqrt{121g^2 + 998g + 121} + 5g - 5}{\sqrt{121g^2 + 998g + 121} - 5g + 5} = \frac{16 + 4g + 8g^2}{14 + 8g^2}.$$

Here are some other theorems.

Theorem

If $2g^3 - 3g^2 - 4 = 0$, *then*

$$\frac{\sqrt{121g^2 + 998g + 121} + 5g - 5}{\sqrt{121g^2 + 998g + 121} - 5g + 5} = \frac{16 + 4g + 8g^2}{14 + 8g^2}.$$

Theorem

If $2g^3 - 3g^2 + 2 = 0$, *then*

$$\frac{\sqrt{1681g^2 - 1882g + 1681} - g + 1}{\sqrt{1681g^2 - 1882g + 1681} + g - 1} = \frac{202 + 144g - 200g^2}{200 + 140g - 200g^2}.$$

Here are some other theorems.

Theorem

If $2g^3 - 3g^2 - 4 = 0$, *then*

$$\frac{\sqrt{121g^2 + 998g + 121} + 5g - 5}{\sqrt{121g^2 + 998g + 121} - 5g + 5} = \frac{16 + 4g + 8g^2}{14 + 8g^2}.$$

Theorem

If $2g^3 - 3g^2 + 2 = 0$, *then*

$$\frac{\sqrt{1681g^2 - 1882g + 1681} - g + 1}{\sqrt{1681g^2 - 1882g + 1681} + g - 1} = \frac{202 + 144g - 200g^2}{200 + 140g - 200g^2}.$$

Can we generate other similar theorems?

References

1. M. D. Hirschhorn and V. Sinescu, *Elementary algebra in Ramanujan's lost notebook*, The Fibonacci Quarterly, **51.2** (2013), 123–129.
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