A GEOMETRIC APPROACH TO A TRIGONOMETRIC INTEGRAL

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An example given to a second semester calculus class consisted of evaluating

$$\int_0^\pi \sin^2 x \ dx \ .$$

The standard ploy used to determine this integral is to first change the integrand by using the trigonometric identity

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x) \ . \tag{1}$$

After carrying out the details to obtain the answer, a student with an apparent phobia for trigonometry inquired as to whether the solution could be obtained without using the half-angle formula (1). The following alternate method yielded an acceptable approach.

By considering the areas of the regions shaded in figures 1 and 2, it is clear that

$$\int_0^\pi \sin^2 x \ dx = \int_0^\pi \cos^2 x \ dx \ . \tag{2}$$

Substituting (2) into the left-hand side of the identity

$$\int_0^\pi (\sin^2 x + cos^2 x) \ dx = \int_0^\pi 1 \ dx = \pi$$

gives

$$2 \int_0^\pi \sin^2 x \ dx = \pi \ ,$$

and hence the desired result after dividing by 2.