## LETTERS TO THE EDITOR

## Dear Editor:

You expressed concern that prospective mathematics teachers may not recognize the relevance of such courses as real analysis and abstract algebra to what they teach. You fear many teachers have the attitude that they only want to be taught that which has obvious application to their classrooms. You recognize that those of us who teach mathematics should be responsible for trying to change such attitudes. But, what should we do, you ask.

I have been teaching mathematics for nearly 30 years, and so have had lots of opportunity to reflect on these issues. All the old people tell me that it's our responsibility to try to pass on the wisdom(?) we have accrued over the years. So, out of respect for my elders, here goes.

I do not believe that most people (let alone prospective teachers who are presumably committed to education, and all that this implies) are opposed to learning. What makes students uncomfortable is being expected to work with ideas they don't understand. Few people are able (and trusting enough) to accurately express their feelings. So, instead they say things like "Why do I need to learn this?" or "This higher math has no relevance to anything I plan to do with my life."

Anyone with an ounce of sensitivity toward teaching children recognizes that any mathematical problems posed to them have to be "concrete". We make problems concrete by drawing pictures, telling stories, providing manipulatives, and, in general, building upon ideas children already have assimilated. Well, adults learn the same way. No one thinks in terms of "abstractions". When you and I do mathematics, we go to great lengths to make the ideas concrete. We draw pictures, examine specific cases, negate hypotheses to see what happens, etc. all in order to get a sense of the "terrain". But most mortals aren't born knowing how to do that. They need to be taught. How many mathematics instructors do you know who spend significant amounts of time with their students just messing around with ideas?

I think that those of us who teach teachers have failed our students. We try to teach prospective mathematics teachers as if they were potential (although slightly retarded) mathematicians. We offer them (revved down?) courses in abstract algebra and real analysis and expect them to be able to take these ideas and make them a part of their mathematical perspective and understanding. And, in the process, we do many of our students a great disservice, for we teach them that mathematics can be a dreadful discipline. We sap them of their enthusiasm for the subject, and, equally important, their self-confidence.

Why do we do this? Maybe partly because we find it overwhelming to think about how to present mathematical ideas in a way that our students can relate to. We begin courses in algebraic structures with an example or two of groups, define groups, and then we're off and running. We notice the agony on our students' faces but can't imagine why it's there. (Are they dumb? Not studying? Lacking in intellectual curiosity?) After all, we do carefully explain the power of generalizing ideas, and as we discuss quotient groups and kernels of homomorphisms we keep referring to specific examples. Our students are silent. They know they are expected to understand, but they don't. They feel stupid, and they know they've let us down.

So, what should we do? Well, we can start by concentrating on our students and how they learn and then adjusting our teaching accordingly. For example, we might offer a course in algebra in which we spend the first two-thirds of the semester playing around with a variety of algebraic structures. (It's great fun solving linear and quadratic equations mod n. How about factoring polynomials over the integers mod 7? Do we have unique factorization? What happens when the mod isn't prime? Ever try to extend the field  $Z_7$  so that ALL quadratics over  $Z_7$  have roots?) There are many specific algebraic structures that are fascinating for students to explore, and, in the process, they begin to get a sense of what it means for an algebraic structure to be nice. That is the time to compare and contrast, and then to generalize.

Unfortunately, one would have to look long and hard to find a textbook that presents algebra in this way. (Does one exist?) Ditto with real analysis, or probability, or (name your own area of mathematics). That's because we, who teach teachers, either lack the time, inclination, or mathematical insight to write materials which are appropriate for our students. (Or is it that publishers wouldn't publish them?) We keep trying to feed our students the same stuff fed us so many years ago. Not that it isn't neat stuff. But, come on now, is it REALLY the appropriate material to teach?

If you ask an interesting question, your students will love working on an answer. The skill in teaching is asking the right questions — ones that your students are ready to work with, and beneath which lie a wealth of ideas — and then helping your students develop the thinking skills that will allow them to become mathematically independent.

By and large, when we are learning  $\cdots$  whatever it is  $\cdots$  we don't question its relevance. It IS relevant because what we learn becomes a part of us and changes us. When our students tell us that what we're teaching is inappropriate for them, believe me, we should believe them!

Sincerely,

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