PROBLEMS

Problems, solutions, and any comments on the problems or solutions should be sent to the problem editor, whose address appears on the inside back cover. An asterisk (*) after a number indicates a problem submitted without a solution.

Problems which are new or interesting old problems which are not well-known may be submitted. They may range from challenging high school math problems to problems from advanced undergraduate or graduate mathematics courses. It is hoped that a wide variety of topics and difficulty levels will encourage a number of readers to actively participate in problems and solutions.

Problems and solutions should be typed or neatly printed on separate sheets of paper. They should include the name of the contributor and the affiliation. Solutions to problems in this issue should be mailed no later than January 15, 1990, although solutions received after that date will also be considered until the time when a solution is published.

13. *Proposed by James H. Taylor, Central Missouri State University, Warrensburg, Missouri.*

(a) Show

$$\sum_{n=0}^{l}\sum_{m=0}^{n}\frac{(-1)^m}{m!(l-n)!}=1$$

for any non–negative integer l.

(b) For any positive integer n, define

$$(2n-1)!!=1\cdot 3\cdot\cdots\cdot(2n-1)$$

and

$$(2n)!!=2\cdot 4\cdot\cdots\cdot(2n)\ .$$

Show

$$\sum_{k=1}^{n}\frac{(2k-1)!!}{(2k)!!}=\frac{(2n+1)!!}{(2n)!!}-1\ .$$

14. *Proposed by Stanley Rabinowitz, Westford, Massachusetts.*

In triangle ABC, AD is an altitude (with D lying on segment BC). DE $\perp$ AC with E lying on AC. X is a point on segment DE such that $\frac{\mathrm{EX}}{\mathrm{XD}} = \frac{\mathrm{BD}}{\mathrm{DC}}$. Prove that AX $\perp$ BE.

15. *Proposed by Leonard L. Palmer, Southeast Missouri State University, Cape Girardeau, Missouri.*

Let F_n denote the n^{th} Fibonacci number ($F_1 = 1$, $F_2 = 1$, and $F_n = F_{n-2} + F_{n-1}$ for $n > 2$) and let L_n denote the n^{th} Lucas number ($L_1 = 1$, $L_2 = 3$, and $L_n = L_{n-2} + L_{n-1}$ for $n > 2$). Find all x such that $F_x + L_x \equiv 0 \pmod 4$ and verify.

16. *Proposed by Mark Ashbaugh, University of Missouri, Columbia, Missouri.*

Consider the $2n \times 2n$ matrix T_n defined as the skew–symmetric matrix for which each entry in the first n subdiagonals below the main diagonal is 1 and each of the remaining entries below the main diagonal is –1. For example,

$$T_1 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

and

$$T_2 = \begin{pmatrix} 0 & -1 & -1 & 1 \\ 1 & 0 & -1 & -1 \\ 1 & 1 & 0 & -1 \\ -1 & 1 & 1 & 0 \end{pmatrix} .$$

Find $\det T_n$ for all positive integers n.