PROBLEMS

Problems, solutions, and any comments on the problems or solutions should be sent to the problem editor, whose address appears on the inside back cover. An asterisk (*) after a number indicates a problem submitted without a solution.

Problems which are new or interesting old problems which are not well–known may be submitted. They may range from challenging high school math problems to problems from advanced undergraduate or graduate mathematics courses. It is hoped that a wide variety of topics and difficulty levels will encourage a number of readers to actively participate in problems and solutions.

Problems and solutions should be typed or neatly printed on separate sheets of paper. They should include the name of the contributor and the affiliation. Solutions to problems in this issue should be mailed no later than January 31, 1991, although solutions received after that date will also be considered until the time when a solution is published.

21. *Proposed by Stanley Rabinowitz, Westford, Massachusetts.*

Find distinct positive integers, a, b, c, d such that

$$\begin{aligned} & a+b+c+d \quad + \quad abcd \\ = & \quad ab+bc+ca+ad+bd+cd \quad + \quad abc+abd+acd+bcd\ . \end{aligned}$$

22. *Proposed by Mohammad K. Azarian, University of Evansville, Evansville, Indiana.*

Without using Riemann sums prove that

$$\lim_{n\to\infty} n^{-3}\left(\sum_{k=1}^{n} k^{\frac{1}{2}}\right)^2 = \frac{4}{9}\ .$$

23. *Proposed by Curtis Cooper and Robert E. Kennedy, Central Missouri State University, Warrensburg, Missouri.*

Let n be a positive integer and $0 \le i \le 9$ be an integer. Let $g_i(n)$ denote the number of digits (base 10) in n which are greater than or equal to i. Prove that

$$\sum_{n=1}^{\infty} \frac{g_2(5^n) + g_4(5^n) + g_6(5^n) + g_8(5^n)}{5^n} = \frac{5}{9} \ .$$

24. *Proposed by Ana Witt (student), Missouri Southern State College, Joplin, Missouri.*

If a linear transformation K on an n–dimensional vector space, V, has $n+1$ eigenvectors such that any n of them span V, what can we say about the linear transformation K?