PROBLEMS

Problems, solutions, and any comments on the problems or solutions should be sent to Curtis Cooper, Department of Mathematics and Computer Science, Central Missouri State University, Warrensburg, MO 64093.

Problems which are new or interesting old problems which are not well-known may be submitted. They may range from challenging high school math problems to problems from advanced undergraduate or graduate mathematics courses. It is hoped that a wide variety of topics and difficulty levels will encourage a number of readers to actively participate in problems and solutions. An asterisk (*) after a number indicates a problem submitted without a solution.

Problems and solutions should be typed or neatly printed on separate sheets of paper. They should include the name of the contributor and the affiliation. Solutions to problems in this issue should be mailed no later than May 31, 1992, although solutions received after that date will also be considered until the time when a solution is published.

37. *Proposed by Russell Euler, Northwest Missouri State University, Maryville, Missouri.*

Lines l_1 and l_2 are concurrent at O. Let $\{a_i\}$ be a sequence of points on l_1 and $\{b_i\}$ be a sequence of points on l_2 such that

$$d(O, a_1) = d(a_i, a_{i+1}) = d(O, b_1) = d(b_i, b_{i+1}) > 0$$

for $i = 1, 2, 3, \ldots$. If M_i is the midpoint of the line segment $\overline{a_i b_i}$, prove that the points M_i are collinear.

38. *Proposed by Stanley Rabinowitz, Westford, Massachusetts.*

Consider the equation: $\sqrt{x_1} + \sqrt{x_2} + \sqrt{x_3} = 0$. Bring the $\sqrt{x_3}$ term to the right-hand side and then square both sides. Then isolate the $\sqrt{x_1 x_2}$ term on one side and square again. The result is a polynomial and we say that we have rationalized the original equation.

Can the equation

$$\sqrt{x_1} + \sqrt{x_2} + \cdots + \sqrt{x_n} = 0$$

be rationalized in a similar manner, by successive transpositions and squarings?

39. *Proposed by Curtis Cooper and Robert E. Kennedy, Central Missouri State University, Warrensburg, Missouri.*

Let n be a positive integer and $L(i)$ denote the number of large digits (digits greater than or equal to 5) in the base 10 representation of the non-negative integer i. Evaluate

$$\frac{1}{10^n} \sum_{i=0}^{10^n-1} L(i)^4 \ .$$

40. *Proposed by Stan Wagon, Macalester College, St. Paul, Minnesota.*

A tetrahedron is a geometric solid with 4 vertices, 6 edges, and 4 triangular faces. A Heron triangle is one whose sides and area are integers. A Heron tetrahedron is one having Heron triangles as faces and whose volume is an integer.

(a) Show that if $\triangle ABC$ is acute, then a tetrahedron exists with each of its faces congruent to $\triangle ABC$.

(b)* John Leech has shown that a Heron tetrahedron exists: Let $\triangle ABC$ have sides 148, 195, and 203 and let T be the tetrahedron obtained from this triangle as in (a). Then each face of T has integer area and T has integer volume. The following question is inspired by Jim Buddenhagen's investigation of Heron triangles whose area is a square. Question: Is there a Heron tetrahedron whose volume is a perfect square or perfect cube?