PROBLEMS

Problems, solutions, and any comments on the problems or solutions should be sent to Curtis Cooper, Department of Mathematics and Computer Science, Central Missouri State University, Warrensburg, MO 64093–5247 or via email to ccooper@cmsuvmb.cmsu.edu.

Problems which are new or interesting old problems which are not well–known may be submitted. They may range from challenging high school math problems to problems from advanced undergraduate or graduate mathematics courses. It is hoped that a wide variety of topics and difficulty levels will encourage a number of readers to actively participate in problems and solutions. An asterisk (*) after a number indicates a problem submitted without a solution.

Problems and solutions should be typed or neatly printed on separate sheets of paper. They should include the name of the contributor and the affiliation. Solutions to problems in this issue should be mailed no later than September 1, 1997, although solutions received after that date will also be considered until the time when a solution is published.

96. *Proposed by Herta T. Freitag, Roanoke, Virginia.*

Let an (m by 2) determinant

$$\det \begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} & \cdots & a_{1,m-1} & a_{1,m} \\ a_{2,1} & a_{2,2} & a_{2,3} & \cdots & a_{2,m-1} & a_{2,m} \end{pmatrix} , \quad m \geq 3$$

be defined by $S(m-1) + \overline{D}$, where

$$S(m-1) = \sum_{r=1}^{m-1} \det \begin{pmatrix} a_{1,r} & a_{1,r+1} \\ a_{2,r} & a_{2,r+1} \end{pmatrix}$$

and

$$\overline{D} = \det \begin{pmatrix} a_{1,m} & a_{1,1} \\ a_{2,m} & a_{2,1} \end{pmatrix}.$$

Consider an (m by 2) determinant D whose elements are polygonal numbers $P_{n,k}$ ($P_{5,3}$ is the 5th triangular number) and such that

$$a_{i,j} = P_{j,i+2},$$

where $i = 1, 2$ and $j = 1, 2, 3, \ldots, m$. Evaluate D.

97. *Proposed by Herta T. Freitag, Roanoke, Virginia.*

Let an (m by 2) determinant

$$\det \begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} & \cdots & a_{1,m-1} & a_{1,m} \\ a_{2,1} & a_{2,2} & a_{2,3} & \cdots & a_{2,m-1} & a_{2,m} \end{pmatrix} , \quad m \geq 3$$

be defined by $S(m-1) + \overline{D}$, where

$$S(m-1) = \sum_{r=1}^{m-1} \det \begin{pmatrix} a_{1,r} & a_{1,r+1} \\ a_{2,r} & a_{2,r+1} \end{pmatrix} \quad \text{and} \quad \overline{D} = \det \begin{pmatrix} a_{1,m} & a_{1,1} \\ a_{2,m} & a_{2,1} \end{pmatrix} .$$

Let

$$D = \det \begin{pmatrix} P_{1,k} & P_{2,k} & P_{3,k} & \cdots & P_{m,k} \\ P_{1,k+a} & P_{2,k+a} & P_{3,k+a} & \cdots & P_{m,k+a} \end{pmatrix} ,$$

where $P_{n,k}$ denotes the nth polygonal number of k "dimensions." For example, $P_{5,3}$ is the 5th triangular number. Evaluate D.

98. *Proposed by Herta T. Freitag, Roanoke, Virginia.*

Let an (m by 2) determinant

$$\det \begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} & \cdots & a_{1,m-1} & a_{1,m} \\ a_{2,1} & a_{2,2} & a_{2,3} & \cdots & a_{2,m-1} & a_{2,m} \end{pmatrix} , \quad m \geq 3$$

be defined by $S(m-1) + \overline{D}$, where

$$S(m-1) = \sum_{r=1}^{m-1} \det \begin{pmatrix} a_{1,r} & a_{1,r+1} \\ a_{2,r} & a_{2,r+1} \end{pmatrix} \quad \text{and} \quad \overline{D} = \det \begin{pmatrix} a_{1,m} & a_{1,1} \\ a_{2,m} & a_{2,1} \end{pmatrix} .$$

Let T_n be the nth triangular number and consider an $((m+2)$ by $2)$ determinant

$$D(m+2) = \det \begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} & \cdots & a_{1,m-1} & a_{1,m} \\ a_{2,1} & a_{2,2} & a_{2,3} & \cdots & a_{2,m-1} & a_{2,m} \end{pmatrix}$$

such that $a_{i,j} = P_{j,i+2}$, where $P_{n,k}$ is the nth polygonal number of k "dimensions" ($P_{3,5}$ is the third pentagonal number). Prove that

$$D(m+2) = \sum_{i=1}^{m} T_i.$$

99. *Proposed by Leonard L. Palmer, Southeast Missouri State University, Cape Girardeau, Missouri.*

For what positive integer values of k is $4k+1$ never a prime?

100*. *Proposed by Bryan Dawson, Emporia State University, Emporia, Kansas.*

Let C be the set of constructible numbers. Let $f: C \to \mathbb{R}$ be given by $f(x) = n$ where n is the minimum number of arcs necessary to construct a segment of length x under the following rules:

1. Only compass and straightedge may be used for the construction.
2. The construction starts with only a segment of unit length, and this segment may not be used for any other purposes than measurement (i.e., the construction cannot be built using the segment; $f(1) = 1$).
3. The number of uses of the straightedge must be finite.

Prove or disprove that f has a point of continuity. æ