HALF–LIFE, BONE LENGTHS, EULER'S FORMULA, AND CONSTRUCTIVISM: WAYS TO INTEGRATE MATHEMATICS AND SCIENCE

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There is a rich historical connection between the development of science and mathematics. Many of the early, notable scientists such as Copernicus, Galileo, and Newton were mathematicians as well as scientists. Farmer, Farrell, and Lehman argue that "modern science cannot exist without mathematics" [7]. Further, they state that mathematical concepts and models are integrated throughout a school's science curriculum. The College Entrance Examination Board describes four levels of science understanding, two of which, the quantitive and symbolic levels of science understanding, rely heavily upon the use of mathematics [7]. Frequently, much of the mathematics found in the middle school science classroom is overlooked or underemphasized. Mathematics teachers can find many hands–on activities that illustrate the usefulness of mathematics in the sciences [e.g., Activities Integrating Mathematics and Science (AIMS) and Great Explorations in Mathematics and Science (GEMS) activity books]. Although many of these activities are traditionally found in the science curriculum, they can provide meaningful applications of mathematical concepts.

National organizations have called for the integration of mathematics and science. The *Curriculum and Evaluation Standards for School Mathematics* states the importance of students being able to "apply mathematical thinking and modeling to solve problems that arise in other disciplines, such as art, music, psychology, science, and business" in grades 5–8 [10]. In addition, the National Research Council in the *National Science Education Standards* argues that, "the science program should be coordinated with the mathematics program to enhance student use and understanding of mathematics in the study of science and to improve student understanding of mathematics" [11].

After reviewing the literature for ways to integrate mathematics and science, we found four encompassing approaches. These approaches include integrating around 1) themes or topics, 2) overlapping concepts or content, 3) common process skills, and 4) common pedagogical skills. Many classroom activities may incorporate more than one of these approaches as illustrated in examples discussed in this article.

The first approach, integrating around a topic or theme, has received a great deal of attention in the literature [5, 6, 9]. Davison, et. al., state that "the topic becomes the medium around which all disciplines interact ... [and that] integration of individual disciplines is subsumed under the integration implied by the investigation of a thematic topic" [5]. Integrating around a theme would not be limited to integrating mathematics and science, but could include most, if not all, subjects found in a middle school.

A unit of study could be built around the theme of nuclear energy. From a mathematics standpoint, one can utilize a widely used activity involving M & M's to investigate the concept of half–life (exponential decay). Using an individual size package of M & M's, students in groups of two place each piece of candy with the M & M logo facing upward; any pieces lacking the logo must not be used in this activity and may be eaten by the students. Students count the number of M & M's with logos and record this information in Table 1 as the number of M & M's at time zero (i.e., before any radioactive decay takes place). Then students place the

Table 1. M & M Half–Life Data Chart

Time (Number of Trials)	Number of M & M's With Logo Up (Undecayed)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

pieces of candy in a small paper cup, shake them, and then pour them out on a table top. Students separate the candies with the logo up from the candies with the logo down and record in Table 1 the number of candies with the "M" up. Students should remove all pieces of the candy with the "M" down (these serve as the decayed portion of our radioactive material), and the teacher may instruct the students that these M & M's may be eaten. Using the pieces of candy with the logo up, students again place them in the cup, shake them, and pour them out. As before, separate the M & M's into two groups, logos up (undecayed) and logos down (decayed). Record the number of M & M's which represent the undecayed material. This process is repeated until all M & M's at some time land logo side down. Using the data, students can construct a graph which should approximate that of a half–life curve. As a whole class activity, students can combine data at each time in Table 1 to arrive at a class data set with its corresponding graph. Not only can a discussion on half–life follow this activity but the class can discuss how one would expect the whole class curve to better approximate a half–life curve than the student group curves. From a science standpoint, one can discuss the concept of half–life, how the half–life of carbon–14 is used to radioactively date certain materials, or other related concepts. In social studies students can report about the history and advantages and disadvantages of nuclear energy, while in a language arts class students can debate such topics as, "Should nuclear waste be stored near their community?" or "Should more nuclear power plants be constructed in the United States?"

According to Berlin and White, "topics common to both science and mathematics curricula include the study of measurement, patterns and relationships, probability and statistics, spatial relationships, and variables and functions"[3]. These content areas readily lend themselves to the integration of mathematics and science. Among the topics that lend themselves to content integration include "natural selection with sampling, randomization, probability, ratio proportionality and area, ... , population growth and genetics with sampling and probability, reflection and refraction with angular geometry" [3]. Other topics are "measurement and dinosaurs, and simple machines and proportions" [5]. For example, biologists can use bone lengths and proportions to estimate the height of an animal. Similarly, middle school students can estimate the height of a classmate by knowing the length of their classmate's femur [15]. In addition to the previously mentioned topics, the M & M half–life activity can provide an example of content integration because half–life is a concept that can be used in a mathematics or science classroom to look at applications of patterns and relationships, and probability and statistics. On the other hand, as mathematics and science teachers collaboratively plan this activity, the mathematics teacher may wish to stress such ideas as simulations, and table and graph construction, while the science teacher emphasizes the concept of half–life. Davison, et. al., argue that the challenge to content integration for mathematics and science teachers is to weave together related objectives from two distinct existing curricula.

Process skills are skills that can be used for learning in any subject area and for life–long learning. These skills help people solve problems and make decisions. The first four NCTM curriculum standards for grades 5–8 are mathematics as 1) problem solving, 2) communication, 3) reasoning, and 4) connections [10]. Many of the skills associated with these four standards are applicable to all content areas. Lists of process skills generally associated with science may vary according to authors, but among those commonly listed are observing, inferring, measuring, communicating, classifying, predicting, controlling variables, forming hypotheses/conjectures, interpreting data, experimenting and formulating models. The importance of these skills is not limited to science but they play a considerable role in the learning, doing, and teaching of mathematics. For example, when integrating around the processes of measuring and observing, one possible activity could include measuring skull photos representing Peking Man to Modern Man and calculating ratios to determine difference in shapes and sizes of skulls [12], while another activity could involve measuring the mass and volume of an object to calculate its density [2]. There are a number of activities that incorporate multiple processes. A good example of this is an activity that investigates Euler's formula using the crystalline forms of minerals which have the shapes of various polyhedra [8]. In this activity, students are required to use process skills that include observing, inferring, communicating, classifying, predicting, forming hypotheses/conjectures, and formulating a model. Students first identify an appropriate mathematical name for the polyhedral shape. Then students observe and record the number of faces, vertices, and edges in a table (see Table 2). Using the information from their tables, students are asked to determine the mathematical relationship among the number of faces, vertices, and edges.

Table 2. Crystals and Euler's Formula

Mineral Name	Polyhedron[1]	No. of Faces	No. of Vertices	No. of Edges
Beryl	hexagonal prism			
Calcite (cleaved)	rhombohedral			
Chalcopyrite	tetrahedron			
Fluorite	octahedron			
Garnet	rhombic dodecahedron			
Pyrite	cube			
Zircon	octadecahedron (18–hedron)			

[1] Many minerals may appear in more than one crystal form, e.g., pyrite and fluorite may appear as an octahedron or a cube.

"*Good* science methodology is integrated in *good* mathematics teaching" [5]. The use of a constructivist model that incorporates the use of discovery and inquiry techniques of teaching is an exemplary pedagogical model for teaching both mathematics and science. In each of the activities mentioned, a constructivist model for learning can be used effectively to obtain the desired outcomes. In particular, the activity where students determine the relationship among the faces, edges, and vertices of crystals, mathematics or science teachers may use guided inquiry techniques to help students discover Euler's formula (# of faces + # of vertices = # of edges + 2). In other words, students work collaboratively to collect the required data to complete their tables. Students are asked to analyze the data they have collected. If they have difficulty discovering Euler's formula, the teacher can ask guiding questions or have students construct additional columns with headings such as "faces + vertices" and "edges + 2" to gain new insight into the problem. Not only do these practices illustrate "good" teaching in mathematics and science, but they provide sound pedagogy for general classroom instruction.

With national standards calling for the integration of school subjects, the opportunity and need for collaboration between mathematics and science teachers has never been greater. As with any change, it is often easier to travel the path with another person, why not a science teacher?

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