PROBLEMS

Problems, solutions, and any comments on the problems or solutions should be sent to Curtis Cooper, Department of Mathematics and Computer Science, Central Missouri State University, Warrensburg, MO 64093 or via email to cnc8851@cmsu2.cmsu.edu.

Problems which are new or interesting old problems which are not well–known may be submitted. They may range from challenging high school math problems to problems from advanced undergraduate or graduate mathematics courses. It is hoped that a wide variety of topics and difficulty levels will encourage a number of readers to actively participate in problems and solutions. An asterisk $(*)$ after a number indicates a problem submitted without a solution.

Problems and solutions should be typed or neatly printed on separate sheets of paper. They should include the name of the contributor and the affiliation. Solutions to problems in this issue should be mailed no later than December 15, 1998, although solutions received after that date will also be considered until the time when a solution is published.

113*. *Proposed by Kamal Jain, Georgia Institute of Technology, Atlanta, Georgia.*

Find all ordered pairs (a, b) such that

$$\tan(a\pi) = b$$

and a and b are rational numbers.

114. *Proposed by Kenneth B. Davenport, P. O. Box 99901, Pittsburg, Pennsylvania.*

(a) Prove that

$$\int_0^\infty \frac{1}{1+x^2} \cdot \frac{4}{4+x^2} \cdot \cdots \cdot \frac{n^2}{n^2+x^2} dx = \frac{\pi}{2} \frac{n}{2n-1}.$$

(b) Prove that

$$\int_0^\infty \frac{1}{1+x^2} \cdot \frac{9}{9+x^2} \cdot \cdots \cdot \frac{(2n+1)^2}{(2n+1)^2+x^2} dx$$

$$= \frac{\pi}{2} \frac{(\Gamma(2n+2))^3}{2^{5n}(2n+1)^3(\Gamma(n+1))^4 \prod_{k=1}^n k(2k-1)}.$$

115. *Proposed by Kenneth B. Davenport, P. O. Box 99901, Pittsburg, Pennsylvania.*

(a) Prove that

The number of ways of expressing every number of the form $3(2n-1)$, $n \geq 1$, as the sum of three numbers is equal to the sum of an nth ranked hexagonal number and an $(n-1)$th square number.

(b) Prove that

The number of ways of expressing every number of the form $4m$, $m \geq 1$, as the sum of four numbers is equal to the sum of the first m tetrahedral numbers, then subtract the sum of the first $m-3$ pentagonal numbers, the first $m-6$ pentagonal numbers, and so on until you reach 0, 1, or 2.

116. *Proposed by Russell Euler and Jawad Sadek, Northwest Missouri State University, Maryville, Missouri.*

Let n be a fixed positive real number and let

$$I_n(t) = \int_0^1 \left[\log\left(\frac{1-rt}{1-r}\right)\right]^n \frac{1-r}{(1-rt)^2} dr,$$

where $0 < t < 1$.

Find an upper bound for $I_n(t)$ as a function of n in terms of the gamma and zeta functions.