PROBLEMS

Problems, solutions, and any comments on the problems or solutions should be sent to Curtis Cooper, Department of Mathematics and Computer Science, Central Missouri State University, Warrensburg, MO 64093 or via email to cnc8851@cmsu2.cmsu.edu.

Problems which are new or interesting old problems which are not well–known may be submitted. They may range from challenging high school math problems to problems from advanced undergraduate or graduate mathematics courses. It is hoped that a wide variety of topics and difficulty levels will encourage a number of readers to actively participate in problems and solutions. An asterisk ($*$) after a number indicates a problem submitted without a solution.

Problems and solutions should be typed or neatly printed on separate sheets of paper. They should include the name of the contributor and the affiliation. Solutions to problems in this issue should be mailed no later than March 15, 1999, although solutions received after that date will also be considered until the time when a solution is published.

117. *Proposed by Kenneth B. Davenport, Pittsburgh, Pennsylvania.*

(a) Prove that

$$\int_0^\infty \frac{1}{1+x^3}\cdot\frac{1}{1+r^3x^3}\cdots\frac{1}{1+(r^nx)^3}dx = \frac{2\pi(1-r^2)(1-r^5)\cdots(1-r^{3n-1})}{3\sqrt{3}(1-r^3)(1-r^6)\cdot(1-r^{3n})}.$$

(b) Prove that

$$\int_0^\infty \frac{1}{1+x^4}\cdot\frac{1}{1+r^4x^4}\cdots\frac{1}{1+(r^nx)^4}dx$$

$$= \frac{\pi}{2\sqrt{2}}\frac{(1-r^3)(1-r^7)\cdots(1-r^{4n-1})}{(1-r^4)(1-r^8)\cdots(1-r^{4n})}.$$

118. *Proposed by Xuming Chen, University of Alabama, Tuscaloosa, Alabama and Kenneth B. Davenport, Pittsburgh, Pennsylvania.*

Let

$$S(a,k,n)=\sum_{i=1}^{n} a^{i-1}i^k.$$

Show that

$$(1-a)S(a,k,n)+\sum_{m=1}^{k}\binom{k}{m}(-1)^m S(a,k-m,n)=-a^n n^k.$$

119. *Proposed by Russell Euler and Jawad Sadek, Northwest Missouri State University, Maryville, Missouri.*

If x and y are distinct positive real numbers, prove that

$$\left(\frac{\sqrt{x}+\sqrt{y}}{2}\right)^2<\frac{5(x+y)^3}{5(x+y)^2+(x-y)^2}<\frac{x-y}{\sinh\left(\frac{x-y}{x+y}\right)}.$$

120. *Proposed by Larry Hoehn, Austin Peay State University, Clarksville, Tennessee.*

Let a, b, and c be the sides of a right triangle such that h is the altitude to hypotenuse c. Prove that $c-2h<a+b<2c-h$.