PROBLEMS

Problems, solutions, and any comments on the problems or solutions should be sent to Curtis Cooper, Department of Mathematics and Computer Science, Central Missouri State University, Warrensburg, MO 64093 or via email to cnc8851@cmsu2.cmsu.edu.

Problems which are new or interesting old problems which are not well–known may be submitted. They may range from challenging high school math problems to problems from advanced undergraduate or graduate mathematics courses. It is hoped that a wide variety of topics and difficulty levels will encourage a number of readers to actively participate in problems and solutions. An asterisk ($*$) after a number indicates a problem submitted without a solution.

Problems and solutions should be typed or neatly printed on separate sheets of paper. They should include the name of the contributor and the affiliation. Solutions to problems in this issue should be mailed no later than December 15, 1999, although solutions received after that date will also be considered until the time when a solution is published.

124. *Proposed by Larry Hoehn, Austin Peay State University, Clarksville, Tennessee.*

Let $\triangle ABC$ be inscribed in a circle and $\triangle A'B'C'$ be circumscribed about the same circle such that the corresponding sides are parallel. Let D', E', and F' be the midpoints of sides $B'C'$, $C'A'$, and $A'B'$, respectively. Prove that AD', BE', and CF' are concurrent. (See figure on next page.)

125. *F. J. Flanigan, San Jose State University, San Jose, California.*

Let d be a positive integer. The real polynomial $h(x)$ is d–simple on the open interval (a, b) if and only if $h(x) = c(x - x_1) \cdots (x - x_e)$ with $0 \leq e \leq d$ and the x_i pairwise distinct elements of (a, b). Let $f(x)$ be a real polynomial of degree $n \geq 1$. Prove that if

$$\int_a^b f(x)h(x)dx = 0$$

for all d–simple polynomails $h(x)$ on (a, b), then

(i) $f(x)$ has at least $d + 1$ roots of odd multiplicity in (a, b), and

(ii) if $d = n - 1$, then $f(x)$ has n distinct simple roots in (a, b).

126. *F. J. Flanigan, San Jose State University, San Jose, California.*

For which coefficient functions $b(t)$ will the equation

$$y''(t) + b(t)y'(t) + \sqrt{1 + y(t)^2} = 0$$

admit a solution $y(t)$ which is oscillatory on some interval, that is, rising and falling repeatedly (as, for example, a perturbed sine wave, or a polynomial with several real roots, etc.)?

127. *Vincent Dunn (student) and Donald P. Skow, University of Texas – Pan American, Edinburg, Texas.*

If x is a triangular number, a and b are positive integers, under what conditions is $ax + b$ also a triangular number. For example, $25x + 3$ satisifies the conditions.

128*. *Curtis Cooper and Robert E. Kennedy, Central Missouri State University, Warrensburg, Missouri.*

Let $b \geq 2$ be a positive integer. Let $g(x)$ be periodic of period $1/(b-1)$ and on $[0, 1/(b-1)]$ be equal to the piecewise linear function connecting the points

$$\left(\frac{d}{b^2 - b}, \frac{(b-d)d}{2b}\right),$$

where d is a nonnegative integer and $0 \leq d \leq b$. Let

$$f(x) = \sum_{i=0}^{\infty} \frac{1}{b^i} g(b^i x).$$

For $b = 2$ it is known that the maximum value of f is $1/3$ and the set E where this occurs is the set of values whose fractional part can be represented as the infinite quaternary fraction $0.\alpha_1\alpha_2 \ldots \alpha_n \ldots$, where every α_i is either one or two.

What is the maximum value of f for a general b and what is the set E where these maximum values occur?