USING GRAPHS TO ANALYZE SPORTS DRAFTS

C. Bryan Dawson

1. Introduction. Sports drafts involving two teams, like any two–player game, can very quickly become complicated to analyze because of the great number of moves that must be considered. Even small examples can cause difficulties. My personal experience with this came when I received a referee's report on the first draft of [1]. Much to my chagrin, I had incorrectly worked an example in the paper. Upon further examination, however, I discovered that the referee's alternative solution was also incorrect! The purpose of this paper is to offer a correct solution to the example with the help of graphs.

First, we need to know the assumptions under which the problem lies. In a general two–team draft, two owners are to pick teams of n players each from a pool of $m \geq 2n$ available players. We do not assume additivity of player values, that is, the value of a team is not necessarily the sum of the values of the individual players; therefore, we will place values on complete teams only. We assume that neither owner knows the other owner's evaluations of the possible teams, and these evaluations may be different. Finally, we assume that the owners will employ a conservative strategy, minimizing risk, rather than a gambling one. It is this last assumption that leads to the difficulties in analysis. For a more complete discussion of these assumptions, see [1].

There are two draft scenarios that we shall consider, leading to two different games. The first scenario is that the two owners will possess the only two teams in existence at the end of the draft. Your goal as an owner is to have a team that you value highly with respect to the other owner's team, i.e., to maximize the quantity

$$\text{your evaluation of your team} - \text{your evaluation of the other owner's team.}$$

The second scenario is that of an "expansion draft," where the two owners are joining a league that already has several other teams. Now, in order to compete with the other teams in the league, your goal as an owner is to maximize your evaluation of your team.

The example we consider is Example 2 from [1].

Example 2. Suppose O_1 and O_2 are to choose three–player teams from among A, B, C, D, E, and F, and the evaluations (e_1, e_2) by O_1 and O_2 of the possible teams are given by:

$\{A,B,C\}-(9,10)$	$\{A,B,D\}-(10,9)$	$\{A,B,E\}-(8,8)$	$\{A,B,F\}-(8,8)$
$\{A,C,D\}-(8,8)$	$\{A,C,E\}-(10,9)$	$\{A,C,F\}-(10,9)$	
$\{A,D,E\}-(8,7)$	$\{A,D,F\}-(8,7)$		
$\{A,E,F\}-(8,7)$			
$\{B,C,D\}-(5,6)$	$\{B,C,E\}-(5,6)$	$\{B,C,F\}-(5,6)$	
$\{B,D,E\}-(6,8)$	$\{B,D,F\}-(6,8)$		
$\{B,E,F\}-(7,9)$			
$\{C,D,E\}-(7,9)$	$\{C,D,F\}-(7,9)$		
$\{C,E,F\}-(6,8)$			
$\{D,E,F\}-(7,10)$			

For the purposes of this paper, we will assume that a "modified draft" (not the "cut–and–choose" draft of [1]) is being used, with O_2 first. The draft order, then, is O_2, O_1, O_1, O_2, O_2, O_1.

2. The Two–Team–Only Case. Who should O_2 choose first? Note that no matter what team O_2 ends up with, the quantity O_2's evaluation of O_2's team – O_2's evaluation of O_1's team is either 1, 0, or -1. Since O_2 goes first, they would certainly hope to be at an advantage, that is, they would hope to make the above quantity 1. Does there exist a strategy to guarantee this? In order to answer the question, we resort to the construction of graphs.

Suppose O_2 chooses A first. Then O_1 will choose the next two players, leaving three players from whom O_2 chooses two. Create a graph with five vertices labeled B, C, D, E, and F. Place an edge between each pair $\{P_1, P_2\}$ of vertices for which $\{A, P_1, P_2\}$ does not result in an advantage for O_2 (i.e., in which the above quantity is 0 or 1). If there is a circuit of length three (i.e., if a triangle exists), then O_1 could choose the two players not in the circuit, meaning that O_2 would be forced to choose two players connected by an edge. Thus, they would not have an advantage. If there is no circuit of length three in the graph, then no matter what two players O_1 chooses, O_2 still is able to pick two remaining players not connected by an edge, and is guaranteed an advantage. By constructing similar graphs for initial choices of B, C, D, E, and F (see Figure 1 at the end of this paper), one can see that O_2 is guaranteed an advantage by choosing A, E, or F first.

Assuming O_2 chooses A first, which two players should be chosen by O_1? Again, graphs come to the rescue. Notice that since O_2 has player A, the quantity O_1's evaluation of O_1's team – O_1's evaluation of O_2's team will be negative. In fact, the possible values for that quantity are -1, -2, -3, and -4. Construct a graph with vertices B, C, D, E, and F, and place an edge between P_1 and P_2 if

$\{A, P_1, P_2\}$ is a team for O_2 for which the quantity above takes on the value -1. If there is a circuit of length three, then O_1 should choose the other two players, for no matter which two of the remaining three players O_2 chooses, O_1 has maximized their quantity at -1. Unfortunately, no such circuit exists (see Figure 2). We then add to the graph edges corresponding to quantities of -2; that is, we add an edge between P_1 and P_2 if $\{A, P_1, P_2\}$ is a team for O_2 for which the quantity above takes on the value -2 (see Figure 2). There is still no circuit of length three, and therefore O_1 does not have a choice that guarantees a quantity of no worse than -2. The edges corresponding to a quantity of -3 are then added. This does result in the presence of circuits of length three. O_1 then chooses either B and C or C and D to maximize their quantity at -3. O_2's choice of two players is then determined by consulting the graph generated in the previous paragraph as explained there. O_1 is given the remaining player and the draft ends.

If O_2 chooses E first, the quantity O_1's evaluation of O_1's team – O_1's evaluation of O_2's team may be as high as 4. We thus begin our graph as in the previous paragraph with edges corresponding to 4, then 3, and so on, continuing until we obtain a circuit of length three. Note that a circuit of length four is present in the graph for quantities 2 or higher, but a circuit of length three is not obtained until the graph for quantities 1 or higher (see Figure 3). At that time, O_1 chooses A and one other player. As before, O_2 chooses the next two players by consulting their graph, and the remaining player goes to O_1. The analysis corresponding to an initial choice of F by O_2 is similar.

3. The Expansion Draft Case. When the objective of a game is changed, optimal strategic play may also change. Such is the case when we change to an expansion draft, leaving all other assumptions of the example unchanged. Recall that now our objective is to obtain a team whose value is maximized.

Again, O_2 is to choose first. The possible values of teams, according to O_2, are 6, 7, 8, 9, and 10. Does O_2 have an initial choice that could guarantee, say, a team worth at least 8? As before, we construct a graph for each of O_2's six possible initial choices. For an initial choice of A, place an edge between P_1 and P_2 if $\{A, P_1, P_2\}$ is a team valued 7 or less. If a circuit of length three exists, O_1 could choose the other two players on their turn, leaving O_2 with a team worth 7 or less. In that case, an initial choice of A would not guarantee O_2 a team worth 8 or more. However, if no such circuit exists, that initial choice would guarantee O_2 a team worth 8 or more. One can see that the only initial choice that cannot guarantee O_2 a team worth 8 is A (see Figure 4).

Now, does there exist an initial choice that can guarantee O_2 a team worth at least 9? We need only add edges to the graphs for initial choices B, C, D, E, and F that correspond to teams valued 8 in the same manner as before. The addition of those edges creates circuits of length three in all graphs except for the initial choice D, whose graph only contains circuits of length four (see Figure 5). Hence, O_2 should choose D first.

Next, O_1 must choose two players. Unfortunately, I have not found graphs helpful in this determination. As O_1 has only ten possible choices of two players, it is not difficult to check the minimum value that each of those choices could give to O_1. The greatest of these minimum values occurs with a choice of A and C, which guarantees O_1 a team worth no less than 9. After consulting their graph, O_2 then chooses E and F, leaving B for O_1. Notice that O_2 actually winds up with a team they value 10, and not just the 9 that they were guaranteed by their initial choice.

4. Conclusion. The astute reader has likely noticed that the above procedures apply directly only to drafts with this particular number of available players and players to be chosen. We leave it as a challenge exercise to expand these methods to drafts with differing number of players available and players chosen. Careful — you just might enjoy yourself!

Reference

1. B. Dawson, "A Better Draft: Fair Division of the Talent Pool," *The College Mathematics Journal*, 28 (1997), 82–88.

C. Bryan Dawson
Department of Mathematics and Computer Science
Union University – Box 1899
1050 Union University Drive
Jackson, TN 38305
email: bdawson@uu.edu