REVIEWS
Edited by Joseph B. Dence

W. Dunham. *Euler: The Master of Us All.* Mathematical Association of America, 1999, pp. 185.

I knew that I would enjoy this book, first because of whom it was about, and second because of by whom it was written. Bill Dunham has surveyed some (surely not all!) of the more than 70 volumes of Euler's *Opera Omnia* (Collected Works), and has selected several juicy tidbits from the long career of this phenomenal mathematician for our enjoyment and amazement. Who cannot be so amazed? Gauss *may* have been more profound, Erdős *may* have been more prolific, but for sheer scope and versatility no one can come close to Euler.

In eight engaging chapters the author sketches some of Euler's contributions in the fields of general number theory, logarithms, infinite series, analytic number theory, complex variables, algebra, geometry, and combinatorics. Some 30 proofs appear in the book. Each chapter concludes with an Epilogue that surveys subsequent developments or mentions related issues that are still unanswered to this day.

Although I was especially interested in the author's two chapters on number theory (my own area), I actually found the chapter on geometry to be the most interesting. The author recounts here, with wit and clarity, Euler's remarkably simple proof of Heron's formula for the area of a triangle,

$$A = \sqrt{s(s-a)(s-b)(s-c)}, \quad s = \frac{a+b+c}{2},$$

and Euler's discovery of the so–called Euler line. Entirely missed by the ancient Greek geometers and by their successors for 2000 years, the theorem of the Euler line says that in any triangle the orthocenter, the centroid, and the circumcenter are collinear, and the centroid is twice as far from the orthocenter as it is from the circumcenter. What a remarkable and beautiful result is this theorem!

Euler's discovery of the Euler line was a tour de force in analytic geometry, which branch of mathematics saw a burst of activity in the 18th century. Euler was, indeed, one of its leading lights. He treated plane coordinate geometry systematically in his two–volume work *Introductio in Analysin Infinitorum* (1748), and to a lesser extent also took up three–dimensional coordinate geometry. Writings such as these, however, led to an intense counterreaction by synthetic geometers early in the 19th century, which Dunham alludes to in his book (see also Chapters 23, 35 in M. Kline, *Mathematical Thought from Ancient to Modern Times*, Oxford University Press, 1972).

If we have dwelled too long on Euler in geometry, it is because there is so much there that is pertinent and useful to students in their first two undergraduate years. Equally useful to undergraduate students is much of Euler's applied work, especially his contributions to mechanics. Unfortunately, none of these have been included in the present book because of spatial limitations. Perhaps applications can form the subject of a future exposition by William Dunham. I'll be in line waiting to buy a copy.

I close with two comments. First, references (other than Euler's) could profitably be gathered together in the rear of the book and also indexed. In the current printing, for example, a reference to Kline as a footnote on page 148 forces one to thumb back through the earlier pages of the book in order to find the original citation; it doesn't appear until page xx! Second, the book is almost free of typographical errors, but on page 146 I would change "Georgonne point" to "Gergonne point" (after Joseph–Diez Gergonne, 1771–1859). The book retails for $29.95, or $23.95 to MAA members. Every departmental library should have a copy.

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