VOCABULARY CONCERNS IN THE MASTERY OF MATHEMATICS: COLLEGE ALGEBRA

Richard L. Francis

1. Introduction. Careless word choices in describing algebraic processes often stem from lack of understanding. Catch–all terms, confusing equations with expressions, or casual terms, suggesting uneasiness about a proper mathematical description, prove frequent in the classroom. Confused labeling, such as the "general quadratic equation" as opposed to the "quadratic formula," has an adverse effect on mathematical perspective as well. All in all, such lack of precision in speech proves revealing in terms of organization of thought and coherence in thinking. Obviously, not all difficulties stem from this source. Yet it is a source deserving of some analysis in the quest for a deeper understanding of College Algebra.

2. A Comparison of Methods. This focus on precision of speech seeks to identify methods of correcting critical deficiencies of word comprehension that may have an adverse effect on College Algebra mastery. Thus, such a study examines foundational terms from elementary algebra that ultimately must support the dialogue and communication efforts of the more advanced mathematical setting. Resolution techniques focus in particular on writing as a powerful tool of clarification, enhancement of understanding, and a paving of the way for more advanced pursuits.

Two sections of College Algebra permitted a comparative look at the benefits and merits of writing activities in the broad area of vocabulary concerns. An early examination first tested all College Algebra students [7] as to their understanding of basic mathematical words from previous courses [6]. Writing activities provided the vocabulary emphasis in section two but only lecture and discussion approaches were utilized in section one. A late examination, testing students in both sections, permitted a comparison of the two groups. Both sections were of random enrollment and essentially and *collectively* of the same level of mathematical maturity.

3. Vocabulary Concerns. Lack of understanding of word meaning may well suggest broader areas of weakness. These in turn identify consequences of a kind that hinders mathematical growth. Select word encounters, so often at odds with precision of speech, prove at first glance quite remarkable because of the exact and quantitative nature of numerical notions.

Areas reflecting the concerns of this study are identified below. They are based on repeated classroom encounters, encounters which range over discussion, dialogue, interaction, student presentation of problems, proofs, derivations, and tests (both diagnostic and achievement).

Thus, vocabulary mastery goes beyond the memorizing and recitation of definitions. Such mastery lies elsewhere and branches out into the realm of an integration of ideas, coherence of expression, and an organization of thought.

Vocabulary concerns below are not necessarily exhaustive. Nor are they to be considered mutually exclusive. Subtleties of connectedness are noted. Likewise, consequences as stated may fail to account fully for the learning gaps and weaknesses that grow out of vocabulary deficiency.

Mathematical Vocabulary: Various Outgrowths

	CONCERN		**CONSEQUENCE**
1.	ignoring of exceptions	1.	wrong generalization
2.	catch–all terminology	2.	failure to account for critical distinctions
3.	casual wording	3.	distortion and actual errors
4.	confused labeling	4.	incorrect positioning of mathematical terms in advanced settings
5.	associating zero with nothing	5.	faulty conclusions
6.	equating sets with proper subsets	6.	theorem misinterpretation
7.	running together of familiar terms	7.	clouded perspective
8.	failure to realize that a mathematical term may have several meanings	8.	weakened understanding of mathematical propositions
9.	superficial classification	9.	ignoring of details critical to a solution
10.	erroneous number classification	10.	confused analysis of mathematical outcomes and violation of context
11.	interchange of descriptions	11.	questionable references
12.	twisted accounts of computational results	12.	extremely slow mastery
13.	inability to describe basic processes	13.	weakening of critical communication

	CONCERN		CONSEQUENCE
14.	non–awareness of great diversity of mathematical expression	14.	narrow patterns of thought
15.	careless treatment of definitions	15.	improper application

Students in each of the two sections of College Algebra were surveyed by a pre–test approach, an approach which established that the conjectured areas of weakness were indeed valid concerns. The objective pre–test was determined by the key mathematical statements of the true–false setting below.

Mathematical Vocabulary — Illustrations

An illustration of basic algebraic mastery concerns in a true–false setting:

1. ignoring of exceptions
   - The absolute value of a real number is not always positive. True
2. catch–all terminology
   - The words "expression" and "equation" may be used interchangeably in mathematics. False
3. casual wording
   - Two negatives give a positive. False
4. confused labeling
   - In reference to $3^2 = 9$, the base is 3 whereas 9 is the power. True
5. associating zero with nothing
   - As zero is the only number satisfying $x + 3 = 3$, the equation has no solution. False
6. equating sets to proper subsets
   - The number labels "complex" and "imaginary" mean the same thing. False
7. running together of familiar terms
   - The statement $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ is called the general quadratic equation. False
8. failure to realize that a mathematical term may have several meanings
   - The number 3 is a zero of $x^2 - 5x + 6$ and is also a root of $x^2 - 5x + 6 = 0$. True
9. superficial classification
   - The equations $2^x = 8$ and $x^2 = 16$ are both referred to as exponential equations as each contains an exponent. False

10.	erroneous number classification	
	• Integers are not necessarily whole numbers.	True
11.	interchange of descriptions	
	• The x–intercept of $2x + 3y = 6$ is the point $(3, 0)$.	False
12.	twisted accounts of computational results	
	• As $5^2 = 25$, it follows that 25 is the square root of 5.	False
13.	inability to describe basic processes	
	• The words "solve," "prove," and "derive" all mean the same thing.	False
14.	non–awareness of great diversity of mathematical expression	
	• The equations $3x + 4y = 12$ and $y = -\frac{3}{4}x + 3$, though different, are actually equivalent.	True
15.	careless treatment of definitions	
	• The discriminant of $ax^2 + bx + c = 0$ is $\sqrt{b^2 - 4ac}$.	False

4. Pre–Test Results. Results of the early pre–test revealed an alarming lack of understanding of the key mathematical vocabulary areas being stressed. Both classes together averaged less than fifty percent in terms of correct responses. The more precise figures are shown in the summary which follows.

Mathematical Vocabulary — A Numerical Look

Vocabulary Mastery: Pre–test (9-29-99)

College Algebra (MA134-06), $n = 36$

Mean = 47.8 (percent)   Standard Deviation = 11.1 (percent)

College Algebra (MA134-07), $n = 32$

Mean = 47.5 (percent)   Standard Deviation = 10.6 (percent)

Classes Combined (MA134-06, MA134-07), $n = 68$

Mean = 47.7 (percent)   Standard Deviation = 10.8 (percent)

5. Post-Test Results. Instructional responses to the results of the pre–test were two–fold. The first section of College Algebra was drilled in the appropriate interpretation of critical vocabulary concerns by the lecture method. The second section was given the assignment of writing correct responses (along with appropriate commentary) to the same vocabulary items. Comparative results follow in the summary below.

Mathematical Vocabulary — A Numerical Look

Vocabulary Mastery: Post–test (11-3-99)

College Algebra (MA134-06), $n = 35$

Mean = 66.0 (percent)   Standard Deviation = 12.9 (percent)

College Algebra (MA134-07), $n = 26$

Mean = 82.0 (percent)   Standard Deviation = 10.5 (percent)

Classes Combined (MA134-06, MA134-07), $n = 61$

Mean = 72.8 (percent)   Standard Deviation = 11.8 (percent)

6. Outcomes. A consequence was considered especially threatening to mathematical mastery if responded to unfavorably by over half the class. They are especially deserving of the teacher's emphasis. Such are referred to as significant areas of vocabulary weakness in the two summaries which follow. One is in reference to the writing class and the other that of the non–writing class. Note that the non–writing class had a post–test average of 66.0 whereas the writing class averaged 82.0.

Significant Areas of Vocabulary Weakness
MA134-07 (Writing Class)

I. Based on Pre–test of 9-29-99

   A. ignoring of exceptions

   B. confused labeling

   C. superficial classification

   D. running together of familiar terms

   E. interchange of descriptions

   F. inability to describe basic processes

   G. careless treatment of definitions

   H. catch–all terminology

II. Based on Post–test of 11-3-99 (following *writing* activity concerning the key vocabulary items)

   A. interchange of descriptions

Significant Areas of Vocabulary Weakness
MA134-06 (Non–writing Class)

I. Based on Pre–test of 9-29-99

   A. ignoring of exceptions

   B. running together of familiar terms

   C. confused labeling

   D. superficial classification

   E. inability to describe basic processes

   F. interchange of descriptions

   G. varied meanings of mathematical terms

   H. catch–all terminology

II. Based on Post–test of 11-3-99 (following class lecture but not writing activity concerning the key vocabulary items)

   A. ignoring of exceptions

   B. inability to describe basic processes

   C. interchange of descriptions

   D. superficial classification

7. Conclusions. Of the fifteen critical areas of vocabulary concern, over half were identified as significant in each of the two College Algebra sections. As noted in the previous list, both classes initially proved weak in the vocabulary areas of the ignoring of exceptions, running together of familiar terms, confused labeling, superficial classification, inability to describe basic processes, interchange of descriptions, and catch–all terminology. Whereas varied meanings of mathematical terms marked the first section only, careless treatment of definitions was significant in the second. These two areas, though different, have similarities that make their inclusion not too surprising.

The non–writing class, after class lecture emphasis, continued to have significant difficulty in the ignoring of exceptions, inability to describe basic processes, interchange of descriptions, and superficial classification. However, the writing class collectively had resolved their difficulties with all of these except the interchange of descriptions.

Areas resolved by a writing emphasis provide a more enlightened setting for mastery. Demands of space preclude a detailed look at each but a sampling of the problem areas reinforces their far–reaching nature. First, consider the ignoring of exceptions and its potential for wrong generalization.

Various topics of College Algebra led to extended probing and frequently culminated in basic conjectures. A major area in which extended probing proves instructionally promising is that of generalization. Yet generalizations must be correct if indeed they are to become a part of the student's storehouse of knowledge. For example, a failure to understand the role of zero in cancellation processes illustrates the matter well. Does $\frac{2x}{3x}$ *always* reduce to $\frac{2}{3}$? Or must $x = 0$ be noted as an exception? Does $ax > b$ *always* imply that $x > \frac{b}{a}$ or must negative and zero values of a be considered exceptions to the rule? Illustrations are abundant of this type of faulty reasoning centering around the meaning of "cancellation" as well as the broad areas of deficiency stemming from the troublesome word "cancel". Other stumbling blocks leading to wrong generalizations concern such familiar terms as "transpose," "cross–multiply," "invert," and "remove parentheses."

Learning by discovery is a major goal in College Algebra (as in all of mathematics) and frequently comes about as a consequence of generalization. Vocabulary deficiency which includes a careless regard for exceptions (e.g., is zero a positive or negative number?) can be corrected by a proper emphasis, an emphasis which can be provided by writing activity. Similar accounts can be given of clouded perspectives and varied harmful outcomes as noted under consequences.

Much is to be gained instructionally by an inventory which focuses on likely error types [2]. This anticipation of student errors can lead to an appropriate writing emphasis. A one–time attempt at resolution by assigned writing may not prove sufficient in all cases. This is evidenced in the study by the persistence of interchange of descriptions, an area of further challenge.

8. Further Explorations. As noted in the introduction, not all difficulties in College Algebra stem from vocabulary weakness. But "word meaning" is an area in which each student can achieve, and moreover provides a critical place of beginning in all mathematical disciplines. Its promising place in College Algebra nevertheless leads to other areas of concern, areas noted in part by the following:

1. What constitutes a satisfactory mathematical vocabulary as the student progresses to the calculus and other more advanced areas of study? Also, what are the major adverse consequences if such vocabulary deficiencies are ignored? [1]
2. If prerequisite vocabulary concerns are addressed by writing activities at a course's beginning (and stressed as their use in teaching occurs), what final

mastery improvements (scores, grades, etc.) can be identified? Interestingly, in this study, out of the 65 students of the combined classes, only two who demonstrated vocabulary mastery (a score of 80 percent or more) received a final grade of deficient.

3. Is retention a monumental problem in College Algebra in other schools and to what extent does "mathematical jargon" (vocabulary) discourage students [4] in the first few days of class?
4. How extensive is the use of casual or imprecise mathematical descriptions in general — and to what extent is the student's basic understanding affected by repeated usage? For example, note such references as "boiling it down" for "simplification," "getting rid of the parentheses" for the "distributive law," and "plugging it in" for "substitution." [5]
5. To what extent do students realize that definitions are reversible and more than simple statements. For example, the true statement "a circle is a closed curve" is not reversible and thus, not a definition.
6. To what extent [3] are "transfer skills" and "capabilities in solving reading problems (applications)" affected by critical vocabulary deficiency?
7. How widespread is the notion that "writing is unimportant in mathematics" as the discipline is essentially a symbolic one? Moreover, how well can the students spell rather than abbreviate key words and use such words in a coherent, accurate, and organized manner?

Even the explorations above do not thoroughly identify the many areas of vocabulary concern and their promise as to success in the teaching–learning situation of mathematics. A certain lack of consistency in terms of definitions ("What is a trapezoid or a rhombus?") remains, not to mention such well–established but confusing references as "1000 is an *odd* power of 10" or "5 is the *imaginary* part of $3 + 5i$." These constitute but the tip of the proverbial iceberg and clearly indicate the need for further research in a critical area of concern in teaching, namely vocabulary and precision of speech.

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Richard L. Francis
Department of Mathematics
Southeast Missouri State University
Cape Girardeau, MO 63701