PROBLEMS

Problems, solutions, and any comments on the problems or solutions should be sent to Curtis Cooper, Department of Mathematics and Computer Science, Central Missouri State University, Warrensburg, MO 64093 or via email to cnc8851@cmsu2.cmsu.edu.

Problems which are new or interesting old problems which are not well–known may be submitted. They may range from challenging high school math problems to problems from advanced undergraduate or graduate mathematics courses. It is hoped that a wide variety of topics and difficulty levels will encourage a number of readers to actively participate in problems and solutions. An asterisk ($*$) after a number indicates a problem submitted without a solution.

Problems and solutions should be typed or neatly printed on separate sheets of paper. They should include the name of the contributor and the affiliation. Solutions to problems in this issue should be mailed no later than January 15, 2004, although solutions received after that date will also be considered until the time when a solution is published.

141. *Proposed by Kenneth B. Davenport, Frackville, Pennsylvania.*

(a) Show that

$$\sum_{k=1}^{n-1} \frac{1}{1-\tan^2\left(\frac{k\pi}{2n}\right)} = \frac{n-1}{2},$$

for $n = 3, 5, 7, 9, \ldots$.

(b) Show that

$$\sum_{k=1}^{n-1} \frac{1}{1+\tan^2\left(\frac{k\pi}{2n}\right)} = \frac{n-1}{2},$$

for $n = 2, 3, 4, 5, \ldots$.

142. *Proposed by Mohammad K. Azarian, University of Evansville, Evansville, Indiana.*

Solve the differential equation

$$y' + \frac{y}{x^x}(x^x \ln y)^n + y(\ln x \ln y) = 0,$$

where n is any real number.

143. *Proposed by José Luis Díaz, Universidad Politécnica de Cataluña, Barcelona, Spain.*

Let α, β, and γ be the angles of triangle ABC. Find the smallest value of

$$\frac{\cot\alpha\cot\beta}{\sqrt{1-\cot\alpha\cot\beta}} + \frac{\cot\beta\cot\gamma}{\sqrt{1-\cot\beta\cot\gamma}} + \frac{\cot\gamma\cot\alpha}{\sqrt{1-\cot\gamma\cot\alpha}}.$$

144. *Proposed by Ovidiu Furdui, Western Michigan University, Kalamazoo, Michigan.*

Prove that in any triangle the following inequality holds:

$$\sum \frac{b+c}{a}\tan\frac{B}{2}\tan\frac{C}{2} \geq 2,$$

where the notations are usual.