PROBLEMS

Problems, solutions, and any comments on the problems or solutions should be sent to Curtis Cooper, Department of Mathematics and Computer Science, Central Missouri State University, Warrensburg, MO 64093 or via email to cnc8851@cmsu2.cmsu.edu.

Problems which are new or interesting old problems which are not well–known may be submitted. They may range from challenging high school math problems to problems from advanced undergraduate or graduate mathematics courses. It is hoped that a wide variety of topics and difficulty levels will encourage a number of readers to actively participate in problems and solutions. An asterisk $(*)$ after a number indicates a problem submitted without a solution.

Problems and solutions should be typed or neatly printed on separate sheets of paper. They should include the name of the contributor and the affiliation. Solutions to problems in this issue should be mailed no later than September 1, 2004, although solutions received after that date will also be considered until the time when a solution is published.

143. *Proposed by José Luis Diaz-Barrero, Universidad Politécnica de Cataluña, Barcelona, Spain.*

Let α, β, and γ be the angles of acute triangle ABC. Prove that

$$\frac{\cot\alpha\cot\beta}{\sqrt{1-\cot\alpha\cot\beta}}+\frac{\cot\beta\cot\gamma}{\sqrt{1-\cot\beta\cot\gamma}}+\frac{\cot\gamma\cot\alpha}{\sqrt{1-\cot\gamma\cot\alpha}}\geq\sqrt{\frac{3}{2}}.$$

149. *Proposed by Joe Howard, Portales, New Mexico and Les Reid, Southwest Missouri State University, Springfield, Missouri.*

Let A, B, C be the angles of a triangle. Show

$$\begin{aligned}3+\cos A+\cos B+\cos C &\geq 2(\sin A\sin B+\sin B\sin C+\sin C\sin A)\\ &\geq 9(\cos A+\cos B+\cos C-1)\end{aligned}$$

with equality if and only if the triangle is equilateral.

150. *Proposed by Ovidui Furdui, Western Michigan University, Kalamazoo, Michigan.*

Evaluate

$$\iint_D \left\{ \frac{1}{x+y} \right\} dx\, dy,$$

where $D = [0,1] \times [0,1]$ and $\{a\}$ is the fractional part of a.

151. *Proposed by José Luis Diaz-Barrero, Universidad Politécnica de Cataluña, Barcelona, Spain.*

Solve the differential equation

$$\sqrt{1+y^2}x^2 e^{\arctan x} dx + \sqrt{1+x^2} dy = 0.$$

152. *Proposed by Joe Flowers and Doug Martin (student), Texas Lutheran University, Seguin, Texas.*

Let

$$F(s) = L[f(t)] = \int_0^\infty e^{-st} f(t)\, dt$$

denote the Laplace transform of $f(t)$. Find $L[\sin^n bt]$, where b is any real constant and n is any non-negative integer.