BOOK REVIEW

Gordon A. Swain

Ian Stewart. *Flatterland: Like Flatland, Only More So.* Perseus Publishing, Cambridge, MA, 2001.

In 1884, Edwin Abbott, a school teacher and Shakespearean scholar, wrote *Flatland* with a dual aim at social satire and mathematical exposition. Though Abbott was not a mathematician, his imagination in describing life in 2-dimensional space and how an additional dimension might be considered by those creatures, created a lasting and influential, though brief work. Since that time, various authors have attempted to write sequels (e.g., in *Sphereland* (Burger, 1965), the author explores curved spaces in a similar style.) or derivative works, the latest of which is the book under review. Ian Stewart has created a masterful tale in the flavor of *Flatland,* but reflecting the actual advances in mathematicians' understanding of geometry, topology, and physical space which have occurred in the last hundred or so years. Stewart is a mathematician, but is able to weave his understanding into a lively visual journey around and into spaces of all types – finite, curved, twisted, and fractal. The witty style brings to mind Douglas Adam's *Hitchhiker's Guide to the Galaxy* series, while paying homage to *Alice in Wonderland.* This book offers much to both the mathematician and the layman. The former will appreciate the inside jokes and plays on words, while seeing, perhaps for the first time, a coordinated presentation of the geometry and topology of classic spaces from a visual and intuitive viewpoint. The latter will make an acquaintance with imaginary worlds, discovering that they may actually live outside the mind of the mathematicians who play with them. By maintaining a mathematical honesty this book provides an antidote to the pseudomathematical gobbledygook that pervades the popular press.

The protagonists of the tale are Vikki (Victoria Line), great-granddaughter of Albert Square (whose adventures were memorialized in *Flatland*), and Space Hopper, the guide who wittily explains each new space in language she can understand. Vikki learns how to untie knots by using an extra dimension, how fractals can have non-integer dimension, meets a cow with only one side of beef (maned Moobius), and watches her left shoe become a right shoe and then back to a left as she takes laps around a projective plane. By looking at spaces both from the outside and the inside (quite a trick) they are able to truly distinguish their essential differences and inherent characteristics.

The style can be seen through a brief conversation between Hopper and Vikki after Hopper has caught a squarrel in Platterland (the hyperbolic plane).

> 'You tell me. Do you *see* anything special?'
>
> Vikki gave the wretched animal a quick once over. 'It's a perfectly ordinary squarrel, just like the ones in Flatland. It's small and furry, square, and it hoards nuts.'
>
> ... 'Are you *sure* it's a square?'
>
> 'Well, admittedly I can't see all its sides without walking round it, but I can recognize right angles when I see them. And when it was running round the tree, all *this* squarrel's angles looked right to me. And all its sides looked the same length, too.'
>
> 'I agree. They are. But did you count it's sides?'
>
> Vikki exploded. 'Now you really are being silly. If all the corners are right angles and all the sides are equal, it's *got* to be a square.'
>
> The Space Hopper grinned. 'Sure?'
>
> 'Absolutely!'
>
> 'OK. Just indulge me. Keep an eye on the animal while I pirouette, and count how many sides it has. You can tell you've got back to the start when its nose comes round again.'
>
> Vikki stamped her vertices in frustration – this was *such* a pointless game. 'Oh, if you insist. What a stupid idea! OK, start turning ... one, two, three, four ... five. See, it's a perfectly ordinary sq – *five?* Hang on, you're cheating. Turn around again. One, two, three, four, five. Again: one, two, three, four, five. Nuts. How are you doing this? This is mad! How can there be a five sided square?'
>
> 'Maybe it's a right-angled pentagon,' the Space Hopper suggested, with a wicked $\cup$.
>
> 'Yeah, maybe – oh, you're awful. That doesn't help at all, it's just as bad. This is crazy.'
>
> The Space Hopper bobbed in agreement, then stopped. 'On the other hand, Vikki, it may just be showing you that Platterland isn't quite as perfectly normal as it appears. After all, it *did* look a bit weird from the vantage of Grand Unified Metaspace.' (pp. 150–1)

This conversation emphasizes in a tangible way that, though from inside it would seem to look just like the Euclidean plane, the hyperbolic plane has intrinsic differences.

Stewart is true to Abbott's aim in extending the conversation to an exploration of the nature of the space we live in, in this case spacetime. After meeting relativistic twins of different ages, I was a bit disappointed at the protracted venture into particle-wave duality, big bang geometry, and string theory. While the progression was logical and smooth, it seemed the author had ventured out of his comfort zone. Nevertheless, the discussion was coherent, understandable, and still amusing. The book finishes with a very interesting contribution to social criticism by means of an enticing explanation of the non-visible differences between men and women.

This book is a thoroughly delightful contribution to the popular literature on mathematical topics which does not lower its standards to make things appear easy; it makes them actually easy through careful exploration, example and analogy. It would be an excellent companion volume for an introductory course in general mathematics (for liberal arts), or could also be used as a starting point for conversation in a seminar with advanced majors. For the student or scholar of mathematics who likes a good read, it is a must.

Gordon A. Swain
Department of Mathematics and Computer Science
Ashland University
Ashland, OH 44805