PROBLEMS

Problems, solutions, and any comments on the problems or solutions should be sent to Curtis Cooper, Department of Mathematics and Computer Science, Central Missouri State University, Warrensburg, MO 64093 or via email to cnc8851@cmsu2.cmsu.edu.

Problems which are new or interesting old problems which are not well–known may be submitted. They may range from challenging high school math problems to problems from advanced undergraduate or graduate mathematics courses. It is hoped that a wide variety of topics and difficulty levels will encourage a number of readers to actively participate in problems and solutions. An asterisk (∗) after a number indicates a problem submitted without a solution.

Problems and solutions should be typed or neatly printed on separate sheets of paper. They should include the name of the contributor and the affiliation. Solutions to problems in this issue should be mailed no later than July 1, 2005, although solutions received after that date will also be considered until the time when a solution is published.

153. *Proposed by Joe Howard, Portales, New Mexico.*

Let $n \geq 2$ be an integer. Prove that

$$n^n > (n+1)^{n-1} + \frac{n}{n+1}.$$

154. *Proposed by Ovidui Furdui, Western Michigan University, Kalamazoo, Michigan.*

Let $n \geq 1$ be an integer and $m > 0$, $m < 2n$. Prove that

$$\int_0^\infty \int_0^\infty \cdots \int_0^\infty \frac{\sin x_1 \sin x_2 \cdots \sin x_n}{(x_1 + x_2 + \cdots + x_n)^m} dx_1 dx_2 \cdots dx_n = \frac{1}{2\Gamma(m)} \cdot B\left(\frac{m}{2}; n - \frac{m}{2}\right),$$

where $\Gamma(\cdot)$ is the Gamma function and $B(\cdot, \cdot)$ is the Beta function.

155. *Proposed by José Luis Díaz-Barrero, Universidad Politècnica de Cataluña, Barcelona, Spain.*

Let P be any point inside $\triangle ABC$ and let $A' = AP \cap BC$, $B' = BP \cap AC$, and $C' = CP \cap AB$. Prove that

$$\frac{1}{AA'^2} + \frac{1}{BB'^2} + \frac{1}{CC'^2} \geq \frac{4\sqrt{3}}{3}\sqrt{\frac{1}{AP^4 + BP^4 + CP^4}}.$$

156. *Proposed by Ovidui Furdui, Western Michigan University, Kalamazoo, Michigan.*

Prove that

$$I = \int_0^{\frac{\pi}{2}} \cos^2 x \left(\ln^2(2\cos x) - \ln(2\cos x)\right) dx = \frac{\pi^3}{48} - \frac{\pi}{4}.$$