SOME GENERALIZATIONS OF PARACOMPACTNESS

Samer Hamed Al Ghour

Abstract. In this paper we introduce the concepts of ω-paracompactness and countable ω-paracompactness as generalizations of paracompactness. We characterize each of them. The study deals with subspaces, products and mappings of each. We conclude this paper with various counterexamples relevant to the relations among the concepts of this paper.

1. Introduction. As defined by Dieudonné [4], a Hausdorff space X is called paracompact if each open covering of X admits a locally finite open refinement. Generalizations of paracompactness have been studied by several authors [1, 5, 10]. The purpose of the present paper is to study some new generalizations of paracompactness, namely ω-paracompactness and countable ω-paracompactness. We characterize each of them and study some of their properties. The study deals with subspaces, products and mappings of each. We conclude this paper with various counterexamples relevant to the relations among the concepts of this paper.

All spaces in this paper are assumed to be T_1. Throughout this paper we follow the notions and conventions of [7]. Now, we list some main definitions and results which will be helpful in obtaining the main results.

<u>Definition 1.1</u>. A point x of a space X is called a condensation point of the set A if any arbitrary neighborhood of the point x contains an uncountable subset of this set.

<u>Definition 1.2</u>. [9] A subset of a space X is called ω-closed if it contains all its condensation points. The complement of an ω-closed set is called ω-open set. Also, if $A \subset X$, then $\underline{A}$ will denote the intersection of all ω-closed sets which contain A.

<u>Definition 1.3</u>. [9] A mapping $f\colon X \to Y$ is called ω-closed if it maps closed sets onto ω-closed sets.

Observe that A is ω-open if and only if for every x in A there is an open set U and a countable subset C such that $x \in U - C \subset A$.

Definition 1.4. Let X be a space and let $A \subset X$. Then we define the ω-interior of A; denoted by $\mathrm{Int}_\omega(A)$; by

$$\mathrm{Int}_\omega(A) = \bigcup \Big\{ B \; : \; B \text{ is } \omega\text{-open in } X \text{ and } B \subset A \Big\}.$$

It is clear that every open set is ω-open. However, the converse is not true. To see this, let $X = \mathbb{R}$ be the set of real numbers with the usual topology. Let A be the set of irrational numbers, then A is ω-open which is not open.

For any topological space X, it is easy to prove the following facts:

(a) The intersection of any two ω-open sets of X is ω-open.
(b) The arbitrary union of ω-open sets of X is ω-open.
(c) If A is a subset of X, then A is ω-closed if and only if $A = \underline{A}$.
(d) If A is a subset of X, then A is ω-open if and only if $A = \mathrm{Int}_\omega(A)$.
(e) If A is a subset of X, then $x \in \underline{A}$ if and only if $U \cap A \neq \emptyset$ for any ω-open set U containing x.

Definition 1.5. [6] A space X is a P-space if the countable intersection of open sets in X is open. A space X is locally countable if every point in X has a countable neighborhood.

Definition 1.6. [3] A family $\{A_\alpha : \alpha \in \Lambda\}$ of subsets of a space X is locally countable if for each $x \in X$ there exists an open neighborhood U such that the family $\{\alpha \in \Lambda : U \cap A_\alpha \neq \emptyset\}$ is countable. A space X is paralindelof if every open cover has a locally countable open refinement.

Definition 1.7. [2] A space X is countably metacompact if every countable open cover of X has a point finite open refinement.

Theorem 1.8. [2] A topological space is countably metacompact if and only if every decreasing sequence $\{F_i\}$ of closed sets with empty intersection has a sequence $\{G_i\}$ of open sets with

$$\bigcap_{i=1}^{\infty} G_i = \emptyset$$

and $G_i \supset F_i$.

2. ω-Local Finiteness, ω-Regularity, and ω-Normality.

Definition 2.1. A family $\{A_\alpha : \alpha \in \Lambda\}$ of subsets of a topological space X is ω-locally finite if for every point $x \in X$ there exists a ω-open set U containing x such that $\{\alpha \in \Lambda : U \cap A_\alpha \neq \emptyset\}$ is finite.

The following result follows directly.

Proposition 2.2. If $\{A_\alpha : \alpha \in \Lambda\}$ is a ω-locally finite family of subsets of a space X, then

$$\underline{\bigcup \{A_\alpha : \alpha \in \Lambda\}} = \bigcup \left\{\underline{A_\alpha} : \alpha \in \Lambda\right\}.$$

From now on, for the space X, $\underset{\sim}{A}$ will denote a family of subsets of X.

Theorem 2.3. If $\underset{\sim}{A}$ is a ω-locally finite family of subsets of a space X, then $\underset{\sim}{A}$ is locally countable.

Proof. Suppose that $\underset{\sim}{A}$ is ω-locally finite and let $x \in X$. Choose an open set U and a countable set C such that $x \in U - C$ and $U - C$ meets at most finitely many members of $\underset{\sim}{A}$. Since $\underset{\sim}{A}$ is point finite, y belongs to finite members of $\underset{\sim}{A}$ for each $y \in C$. So $(U - C) \cup C$ must meet at most countably many members of $\underset{\sim}{A}$. The result follows since $U \subset (U - C) \cup C$.

Theorem 2.4. Let X be a space and let $\underset{\sim}{A}$ be a family of subsets of X.

(i) If X is P-space then $\underset{\sim}{A}$ is locally finite if and only if $\underset{\sim}{A}$ is ω-locally finite.

(ii) If X is a locally countable space, then $\underset{\sim}{A}$ is ω-locally finite if and only if $\underset{\sim}{A}$ is point finite.

Proof. The proof is straightforward.

A mapping $f\colon X \to Y$ is finite to one if $f^{-1}(y)$ is finite for every $y \in Y$.

Theorem 2.5. Let $f\colon X \to Y$ be a finite to one ω- closed mapping from X onto Y. If $\underset{\sim}{A}$ is ω-locally finite in X, then $f(\underset{\sim}{A})$ is ω-locally finite in Y.

Proof. Let $y \in Y$, then $f^{-1}(y)$ is a nonempty finite say, $f^{-1}(y) = \{x_1, x_2, \dots, x_n\}$. If $\underset{\sim}{A}$ is ω-locally finite, then for each $1 \leq i \leq n$ there exists an open set U_i and a countable subset C_i such that $x \in U_i - C_i$ and $U_i - C_i$ meets at most finitely many members of $\underset{\sim}{A}$. Let

$$G = \left(\bigcup_{i=1}^{n} U_i\right) - \bigcup_{i=1}^{n} (C_i - D_i),$$

where $D_i = f^{-1}(y) - \{x_i\}$, then G meets at most finitely many members of $\underset{\sim}{A}$ and $f^{-1}(y) \subset G$. Now $V = Y - f(X - G)$ is ω-open, moreover, $y \in V$. Finally, if $V \cap f(A) \neq \emptyset$ for some $A \in \underset{\sim}{A}$, pick $a \in A$ such that $f(a) \in V$, then $a \in f^{-1}(V) = X - f^{-1}(f(X - G)) \subset X - (X - G) = G$. Thus, $A \cap G \neq \emptyset$. Hence, V meets at most finitely many members of $f(\underset{\sim}{A})$. Therefore, $f(\underset{\sim}{A})$ is ω-locally finite.

Theorem 2.6. Let X be a topological space with the property that every nonempty open set is uncountable. If A is ω-open in X then $\underline{A} = \overline{A}$.

Proof. Since the result is obvious whenever $A = \emptyset$, we may assume $A \neq \emptyset$. Let $x \in \overline{A}$ and let U be any ω-open subset of X containing x. Let V_1 be an open set in X and C_1 be a countable subset of X such that $x \in V_1 - C_1 \subset U$. Since $x \in V_1 \cap \overline{A}, V_1 \cap A \neq \emptyset$. Let V_2 be an open set in X and C_2 be a countable subset of X such that $V_2 - C_2 \subset A$ and $V_1 \cap (V_2 - C_2) \neq \emptyset$. From the assumption $V_1 \cap V_2 \neq \emptyset$ is uncountable. Thus, $(V_1 - C_1) \cap (V_2 - C_2) \neq \emptyset$. Therefore, $U \cap A \neq \emptyset$ and hence, $x \in \underline{A}$.

Conversely, clearly that $\overline{A} \subset \underline{A}$.

Definition 2.7. A space X is said to be

(i) ω-regular if whenever F is closed in X and $x \in X - F$, then there are disjoint ω-open and open sets U and V, respectively with $x \in U$ and $F \subset V$.

(ii) ω-normal if whenever A and B are disjoint closed sets in X, there are disjoint open and ω-open sets U and V, respectively with $A \subset U$ and $B \subset V$.

The following characterization of ω-normal spaces follows easily.

Theorem 2.8. A space X is ω-normal if and only if whenever A is closed in X and U is open in X with $A \subset U$, then there is an open set V in X such that $A \subset V \subset \underline{V} \subset U$.

Corollary 2.9. If X is a space with the property that every nonempty open set is uncountable then X is normal if and only if X is ω-normal.

3. ω-Paracompact Spaces.

Definition 3.1. A topological space X is called an ω-paracompact space if X is a Hausdorff space and every open cover of X has an ω-locally finite open refinement.

Remark 3.2. Paracompact $\Rightarrow$ ω-paracompact $\Rightarrow$ metacompact.

In the last section, we shall see that the converse of each of the above implications is not true. However, we have the following consequences of Theorem 2.4.

Theorem 3.3. Let X be a P-space. Then the following are equivalent.

(i) X is paracompact.

(ii) X is ω-paracompact.

Theorem 3.4. Let X be a locally countable space. Then the following are equivalent.

(i) X is ω-paracompact.

(ii) X is metacompact.

Although paracompact spaces are normal, we shall see later on that ω-paracompact spaces are not even regular. However, we have the following result.

Theorem 3.5. Every ω-paracompact space is ω-regular.

Proof. Let X be an ω-paracompact space. Let F be a closed subset of X and let $x \in X - F$. Since X is Hausdorff we can choose for every $y \in F$ open set V_y containing y such that $x \notin \overline{V_y}$. Then $\{V_y \ : \ y \in F\} \cup \{X - F\}$ is an open cover of X and thus has an ω-locally finite open refinement $\underset{\sim}{W}$. Take

$$V = \bigcup \left\{ W \in \underset{\sim}{W} : W \cap F \neq \emptyset \right\},$$

then V is open in X and by Proposition 2.2,

$$\underline{V} = \bigcup \left\{ \underline{W} \in \underset{\sim}{W} : W \cap F \neq \emptyset \right\}.$$

Now $F \subset V$, $x \in U$, and $U \cap V = \emptyset$. Therefore, X is ω-regular.

Question 3.6. Is every ω-paracompact space ω-normal?

Theorem 3.7. If X is ω-paracompact with the property that every nonempty open set is uncountable, then X is normal.

Proof. Using Theorem 2.6 and a proof similar to that used in Theorem 3.5 we can conclude that X is regular.

Now, suppose A and B are disjoint closed sets in X. For each $y \in A$, by regularity, find open V_y such that $y \in V_y$ and $\overline{V_y} \cap B = \emptyset$. Now using ω-paracompactness and Theorem 2.6 we can produce an open set V such that $A \subset V$ and $\overline{V} \cap B = \emptyset$. Thus, X is normal.

The following relation between ω-paracompact and paralindelof follows from Theorem 2.3.

Theorem 3.8. Every ω-paracompact space is paralindelof.

Question 3.9. Is every paralindelof space ω-paracompact?

4. Countably ω-Paracompact Spaces.

Definition 4.1. A Hausdorff space X is countably ω-paracompact if every countable open cover has an ω-locally finite open refinement.

Remark 4.2. Countably paracompact $\Rightarrow$ countably $\omega-$paracompact $\Rightarrow$ countably metacompact.

It will be seen that the converse of each of the above implications is not true in general. The following results follow from Theorem 2.4.

Theorem 4.3. Let X be a P-space. Then the following are equivalent.
(i) X is countably paracompact.
(ii) X is countably ω-paracompact.

Theorem 4.4. Let X be a locally countable space. Then the following are equivalent.
(i) X is countably ω-paracompact.
(ii) X is countably metacompact.

Theorem 4.5. For every Hausdorff space X the following are equivalent.
(i) The space X is countably ω-paracompact.
(ii) For every countable open cover $\{U_i\}_{i=1}^{\infty}$ of the space X there exists an ω-locally finite open cover $\{V_i\}_{i=1}^{\infty}$ of X such that $V_i \subset U_i$ for $i = 1, 2, \ldots$.
(iii) For every increasing sequence $W_1 \subset W_2 \subset \ldots$ of open subsets of X satisfying

$$\bigcup_{i=1}^{\infty} W_i = X,$$

there exists a sequence $F_1, F_2, \ldots$ of closed subsets of X such that $F_i \subset W_i$ for $i = 1, 2, \ldots$ and

$$\bigcup_{i=1}^{\infty} \mathrm{Int}_{\omega}(F_i) = X.$$

(iv) For every decreasing sequence $F_1 \supset F_2 \supset \dots$ of closed subsets of X satisfying

$$\bigcap_{i=1}^{\infty} F_i = \emptyset,$$

there exists a sequence $W_1, W_2, \dots$ of open subsets of X such that $F_i \subset W_i$ for $i = 1, 2, \dots$ and

$$\bigcap_{i=1}^{\infty} \underline{W_i} = \emptyset.$$

Proof. (i) $\Rightarrow$ (ii) Let $\{U_i\}_{i=1}^{\infty}$ be any countable open cover for X. Let $\underset{\sim}{V}$ be an open ω-locally finite refinement of $\{U_i\}_{i=1}^{\infty}$, for every $V \in \underset{\sim}{V}$ choose a natural number $i(V)$ such that $V \subset U_{i(V)}$, and let $V_i = \bigcup \{V : i(V) = i\}$. Then $\{V_i\}_{i=1}^{\infty}$ is an open ω-locally finite cover of X with $V_i \subset U_i$ for $i = 1, 2, \dots$.

(ii) $\Rightarrow$ (iii) Let $W_1 \subset W_2 \subset \cdots$ be an increasing sequence of open subsets of X such that

$$\bigcup_{i=1}^{\infty} W_i = X.$$

By (ii), there exists an ω-locally finite open cover $\{V_i\}_{i=1}^{\infty}$ of X such that $V_i \subset W_i$ for $i = 1, 2, \dots$, let

$$F_i = X - \bigcup_{j>i} V_j,$$

then F_i is closed and

$$F_i \subset \bigcup_{j \le i} V_j \subset \bigcup_{j \le i} W_j = W_i$$

for $i = 1, 2, \dots$. Let $x \in X$. Since $\{V_i\}_{i=1}^{\infty}$ is ω-locally finite, there exists an ω-open set U_x containing x which meets at most finitely many members of

$\{V_i\}_{i=1}^{\infty}$ say, $V_{i_1}, V_{i_2}, \dots, V_{i_m}$. Let $t = \max\{i_1, i_2, \dots, i_m\}$, then $U_x \subset F_t$. Thus, $x \in \mathrm{Int}_\omega(F_t)$ and hence,

$$\bigcup_{i=1}^{\infty} \mathrm{Int}_\omega(F_i) = X.$$

(iii) $\Rightarrow$ (iv) Let $F_1 \supset F_2 \supset \cdots$ be any decreasing sequence of closed subsets of X satisfying

$$\bigcap_{i=1}^{\infty} F_i = \emptyset,$$

then $X - F_1 \subset X - F_2 \subset \cdots$ and

$$\bigcup_{i=1}^{\infty} (X - F_i) = X,$$

so by (iii) there exists a sequence $M_1, M_2, \dots$ of closed subsets of X such that $M_i \subset X - F_i$ for each i and

$$\bigcup_{i=1}^{\infty} \mathrm{Int}_\omega(M_i) = X.$$

Let $W_i = X - M_i$, then W_i is open and $F_i \subset W_i$ for each i. Moreover, since $\underline{X - M_i} = X - \mathrm{Int}_\omega(M_i)$ for each i and

$$\bigcup_{i=1}^{\infty} \mathrm{Int}_\omega(M_i) = X$$

we get

$$\bigcap_{i=1}^{\infty} \underline{W_i} = \emptyset.$$

(vi) $\Rightarrow$ (iii) Mimic the proof of (iii) $\Rightarrow$ (iv).

(iii) $\Rightarrow$ (i) Let $\{U_i\}_{i=1}^{\infty}$ be any countable open cover for X. Consider the increasing sequence $W_1 \subset W_2 \subset \dots$ of open subsets of X, where

$$W_i = \bigcup_{j \leq i} U_j.$$

Since

$$\bigcup_{i=1}^{\infty} W_i = X,$$

there exists a sequence $F_1, F_2, \dots$ of closed subsets of X such that $F_i \subset W_i$ for $i = 1, 2, \dots$ and

$$\bigcup_{i=1}^{\infty} \mathrm{Int}_{\omega}(F_i) = X.$$

The set

$$V_i = U_i - \bigcup_{j<i} F_j \subset U_i$$

is open for $i = 1, 2, \dots$ since

$$\bigcup_{j<i} F_j \subset \bigcup_{j<i} W_j \subset \bigcup_{j<i} U_j,$$

it follows that

$$U_i - \bigcup_{j<i} U_j \subset V_i$$

and hence, the family $\{V_i\}_{i=1}^{\infty}$ is a cover of X. Every point $x \in X$ is contained in an ω-open set of the form $\mathrm{Int}_{\omega}(F_j)$ with $\mathrm{Int}_{\omega}(F_j) \cap V_i = \emptyset$ for all $i > j$. Therefore, the cover $\{V_i\}_{i=1}^{\infty}$ is ω-locally finite.

Corollary 4.6. Let X be a Hausdorff space with the property that every nonempty open subset of X is uncountable then X is countably paracompact if and only if X is countably ω-paracompact.

Theorem 4.7. For every ω-normal space X the following are equivalent.

(i) X is countably ω-paracompact.

(ii) X is countably metacompact.

Proof. (i) $\Rightarrow$ (ii) The proof is obvious.

(ii) $\Rightarrow$ (i) Suppose X is ω-normal countably metacompact space and let $F_1 \supset F_2 \supset \cdots$ be a decreasing sequence of closed subsets of X with

$$\bigcap_{i=1}^{\infty} F_i = \emptyset.$$

Since X is countably metacompact, then by Theorem 1.8, it follows that there exists a sequence $S_1, S_2, \ldots$ of open subsets of X such that $F_i \subset S_i$ for $i = 1, 2, \ldots$ and

$$\bigcap_{i=1}^{\infty} S_i = \emptyset.$$

Since X is ω-normal, for each i, we pick an open subset W_i of X such that $F_i \subset W_i \subset \underline{W_i} \subset S_i$. Since

$$\bigcap_{i=1}^{\infty} S_i = \emptyset \ , \ \bigcap_{i=1}^{\infty} \underline{W_i} = \emptyset.$$

Therefore, by Theorem 4.5, X is countably ω-paracompact.

5. Subspaces, Maps and Products.

The following result concerning subspaces can be obtained easily.

Theorem 5.1.

(i) Each of ω-paracompactness and countable ω-paracompactness is hereditary with respect to closed subspaces.

(ii) If every open subspace of the topological space X is ω-paracompact (countably ω-paracompact) then every subspace of X is ω-paracompact (countably ω-paracompact).

Question 5.2. Is ω-paracompactness (countable ω-paracompactness) hereditary with respect to F_σ sets?

Hanai [8] proved that paracompactness and countable paracompactness are inverse invariant under perfect mappings. In the last section, we shall see that each of ω-paracompactness and countable ω-paracompactness is not inverse invariant under perfect mappings.

Definition 5.3. A function $f\colon X \to Y$ is said to be strongly continuous if f is continuous and $f^{-1}(U)$ is ω-open whenever U is ω-open. A strongly perfect map is just a strongly continuous map which is perfect.

Theorem 5.4. Let X be a Hausdorff space and let $f\colon X \to Y$ be a strongly perfect map from X onto Y. If Y is ω-paracompact then so is X.

Proof. Let Y be ω-paracompact and let $\underset{\sim}{U}$ be an open cover of X. For each $y \in Y$, let $\underset{\sim}{U}_y$ be a finite subcollection of $\underset{\sim}{U}$ that covers $f^{-1}(y)$, for

every $y \in Y$, choose an open neighborhood V_y of y such that

$$f^{-1}(V_y) \subset U_y = \bigcup \left\{ U : U \in \underset{\sim}{U}_y \right\}.$$

Then $\underset{\sim}{V} = \{V_y : y \in Y\}$ is an open cover of the ω-paracompact space Y and so it has an ω-locally finite open refinement $\underset{\sim}{W}$. Take

$$\underset{\sim}{B} = \left\{ f^{-1}(W) : W \in \underset{\sim}{W} \right\}.$$

Then $\underset{\sim}{B}$ is ω-locally finite open cover of X.

For each $W \in \underset{\sim}{W}$ choose a point $y_W \in Y$ such that $W \subset V_{y_W}$.
Take

$$\underset{\sim}{A} = \left\{ f^{-1}(W) \cap U : U \in \underset{\sim}{U}_{y_W}, W \in \underset{\sim}{W} \right\}.$$

Then $\underset{\sim}{A}$ is an ω-locally finite open refinement of $\underset{\sim}{U}$. Therefore, X ω-paracompact.

Corollary 5.5. If X is compact and countable and Y is ω- paracompact then $X \times Y$ is ω-paracompact.

The proof of the following result follows using a method similar to that used in Theorem 5.4.

Theorem 5.6. If f is a strongly continuous closed map of a Hausdorff space X onto a countably ω-paracompact space Y such that the inverse image $f^{-1}(y)$ is countably compact for every point $y \in Y$ then X is countably ω-paracompact.

Corollary 5.7. If X is compact and countable and Y is countably ω-paracompact then $X \times Y$ is countably ω-paracompact.

6. Counter Examples.

In this section, we introduce three examples which answer the converse of many implications and many questions throughout this paper.

Example 6.1. [10] Let ω_1 denote the first uncountable ordinal and let $[0, \omega_1]$ be the space of all ordinal $\alpha \leq \omega_1$ endowed with the order topology. This example gives us that countable ω-paracompactness does not imply ω-paracompactness, ω-paracompactness is not hereditary under open subspaces and ω-normal spaces need not be ω-paracompact spaces.

Example 6.2. [11] The irrational slope topology with the set $X = \{(x, y) : y \geq 0, x, y \text{ are rationals}\}$ gives us that ω-paracompactness does not imply countable paracompactness, ω-paracompactness does not imply paracompactness, ω-paracompactness does not imply regularity, ω-normality does not imply regularity and countable ω-paracompactness does not imply countable paracompactness.

Hanai [8] proved that in the realm of Hausdorff spaces, countable paracompactness is invariant of closed continuous mappings with countably compact fibers. This result is useful to study the following example.

Example 6.3. Let $Y = [0, 1] \times X$ be the product space of the subspace $[0, 1]$ of the usual topology and the irrational slope topology X. If Y is ω-normal then by Corollary 2.9, it must be normal, thus X is normal which is not true. If Y is countably ω-paracompact then by Corollary 4.6, it will be countably paracompact. Therefore, X must be countably paracompact which is not true.

From the previous example we conclude the following.

1. Metacompactness does not imply ω-paracompactness.
2. Countable metacompactness does not imply countable ω-paracompactness.
3. The product of two ω-normal spaces one of which is compact and metric need not be ω-normal.

4. The product of two countably ω-paracompact spaces one of which is compact and metric need not be countably ω-paracompact.
5. The product of two ω-paracompact spaces one of which is compact and metric need not be ω-paracompact.
6. Countable ω-paracompactness need not be inverse invariant of perfect open mappings.
7. ω-paracompactness need not be inverse invariant under perfect open mappings.
8. Continuity does not imply strong continuity.
9. Perfectness of mappings does not imply strong perfectness.

At the end of this section we raised the following question.

Question 6.4. Is every ω-paracompact regular space paracompact?

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Samer Hamed Al Ghour
Department of Mathematics and Statistics
Jordan University of Science and Technology
Irbid 22110, Jordan
email: algore@just.edu.jo